Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational and Applied Mathematics 222 (2008) 392–403
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
www.elsevier.com/locate/cam

Group analysis for natural convection from a vertical plate

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Received 18 May 2007; received in revised form 5 November 2007

Abstract

The steady laminar natural convection of a fluid having chemical reaction of order n past a semi-infinite vertical plate is considered. The solution of the problem by means of one-parameter group method reduces the number of independent variables by one leading to a system of nonlinear ordinary differential equations. Two different similarity transformations are found. In each case the set of differential equations are solved numerically using Runge–Kutta and the shooting method. For each transformation different Schmidt numbers and chemical reaction orders are tested.

MSC: 76R10; 35Q53; 35B20

Keywords: Chemical diffusion; Natural convection; Group method; Schmidt number

1. Introduction

Natural convection flow is the fluid motion due to buoyancy forces depending on the fluid density gradient and gravitational force. Such flows have wide band applications in different engineering fields and technological processes such as the chemical coating of flat surfaces.

The problem of steady-state natural convection induced by chemical diffusion from a vertical plate was reported by Levich [9] in 1962. This plate is immersed in a fluid solution having a concentration $c_0(x) > 0$. When the plate touches the solution, a chemical reaction takes place inducing a change of concentration and implying density gradients in the presence of gravitational field. This problem was investigated by Gebhart et al. [7] and Mulolani et al. [12] through Birkhof's [5] transformation method. In 2002 Muthukumaraswamy et al. [13] proposed a numerical solution of a first-order homogeneous chemical reaction. A further investigation of the Schmidt number effect on the reaction rate and mass diffusion process was carried out by Rahman et al. [16] through perturbation expansions about an additional similarity variable depending on the concentration. Recently Postelnicu [17] numerically investigated the characteristics of natural convection of a vertical surface.

The mathematical technique used in the present analysis is a group transformation of the variables, leading to a similarity representation of the problem. In this method developed by Morgan [11] a group is assumed; and consequently, the general form of the invariants is deduced. The requirement of invariance of equations under the

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doi:10.1016/j.cam.2007.11.010

Nomenclature

Latin characters

a	group unity element
c	dimensional species concentration
c_0	concentration next to the vertical plate
c_∞	concentration in the ambient fluid
D	chemical molecular diffusivity
F	vertical velocity after transformation
g	gravitational acceleration
G	group
k	chemical rate constant
m	aspect transformation ratio of y with respect to x for case 1
n	order of chemical reaction
Q, T	real-valued coefficients
r	aspect transformation ratio of y with respect to x for case 2
S	subgroup
Sc	Schmidt number ν/D
u	velocity in x direction
v	velocity in y direction
x	vertical distance
y	horizontal distance from the plate surface

Greek characters

β	volumetric coefficient of expansion with concentration
$\nu = \mu/\rho$	kinematic viscosity of fluid
ρ	fluid density
ψ	stream function
η	similarity variable

assumed group generates a set of simple simultaneous equations whose solution determines the specific form for the invariants see [6,10]. This method adopted by Abd El Malek proved [1] to be efficient [2–4,14,15] for the analysis of various boundary layer flow problems.

In the present work, we provide an analytical and numerical solution for natural convection induced by a chemical reaction adjacent to a vertical plate, using the group transformation method. Under the application of one-parameter group, the governing partial differential equations and boundary conditions are reduced to ordinary differential equations with appropriate boundary conditions. The obtained differential equations are solved numerically using the shooting method.

2. Mathematical formulation

We are in the present study concerned with the convection and diffusion within a thin boundary layer adjacent to the vertical plate immersed in a fluid having a chemical reaction of order n. For this, it is convenient to consider an idealized system composed of a semi-infinite plate set in a fluid of infinite extent. The natural convection from a vertical plate is described by the equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - g\beta(c - c_\infty) - \nu\frac{\partial^2 u}{\partial y^2} = 0 \tag{2.2}$$