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A variable step implicit block multistep method for solving first-order ODEs

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ABSTRACT

A new four-point implicit block multistep method is developed for solving systems of first-order ordinary differential equations with variable step size. The method computes the numerical solution at four equally spaced points simultaneously. The stability of the proposed method is investigated. The Gauss–Seidel approach is used for the implementation of the proposed method in the $PE(CE)^m$ mode. The method is presented in a simple form of Adams type and all coefficients are stored in the code in order to avoid the calculation of divided difference and integration coefficients. Numerical examples are given to illustrate the efficiency of the proposed method.

1. Introduction

In many fields of application in, e.g., science and engineering one can find equations of the form

$$Y' = F(x, Y) \qquad Y(a) = Y_0, \quad a \le x \le b \tag{1}$$

where a and b are finite and $Y' = [y'_1, y'_2, \ldots, y'_n]^T$, $Y = [y_1, y_2, \ldots, y_n]^T$ and $F = [f_1, f_2, \ldots, f_n]^T$. Most of the existing methods for solving ODEs like that in (1) will only approximate the numerical solutions at one point, sequentially. Thus, developing faster methods which can give faster solutions to the problem are needed.

Block methods for the numerical solution of first-order ODEs have been proposed by several authors, such as [1–4]. Among the earliest researchers investigating the block method, Houwen and Sommeijer [5] have developed block Runge–Kutta methods, Omar [6] introduced a block method based on Adams formulas for solving higher order ODEs and Majid [7] proposed a variable step size and order Adams type block method. The advantage of a block method is that in each application, the solution will be approximated at more than one point. The number of points depends on the structure of the block method. Therefore, applying these methods can give faster solutions to the problem and also can be managed to produce a desired accuracy.

The authors in [8,9] have introduced a four-point diagonally and fully implicit block method in which at each application of the method, the solution will be approximated at four points simultaneously. The Jacobi iteration was used for the implementation of the methods in [8,9].

The Gauss–Seidel approach for the implementation of the two-point block one-step method was discussed in [10]. In this paper, the same approach will be considered for the four-point implicit block multistep method. The proposed block method will approximate the solutions at four points simultaneously in each step, using variable step size.

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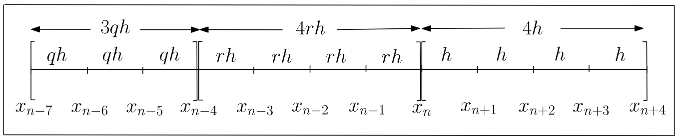

Fig. 1. Four-point implicit block multistep method.

The method is derived by using the Lagrange interpolation polynomial and the closest point in the interval will be considered for obtaining the corrector and predictor formula. Therefore, the approximated values of y_{n+1}, y_{n+2}, y_{n+3} and y_{n+4} are obtained by integrating (1) over the intervals $[x_n, x_{n+1}]$, $[x_{n+1}, x_{n+2}]$, $[x_{n+2}, x_{n+3}]$ and $[x_{n+3}, x_{n+4}]$ respectively.

2. Derivation of the four-point implicit block multistep method

In Fig. 1, the solutions for $y_{n+1}, y_{n+2}, y_{n+3}$ and y_{n+4} with step size h at the points $x_{n+1}, x_{n+2}, x_{n+3}$ and x_{n+4} respectively are approximated simultaneously using five back values at the points $x_n, x_{n-1}, x_{n-2}, x_{n-3}$ and x_{n-4} of the previous four steps with step size rh. The set of points $\{x_{n-7}, \dots, x_n\}$ are used for deriving the predictor formula and the order is 1 less than the order of the corrector. The method will compute the solution at four points concurrently using four earlier steps.

The interpolation points involved for obtaining the corrector formulas for $y_{n+1}, y_{n+2}, y_{n+3}$ and y_{n+4} are $\{(x_{n-4}, f_{n-4}), \dots, (x_{n+4}, f_{n+4})\}$. The first point y_{n+1} is derived by integrating (1) as follows:

$$\int_{x_n}^{x_{n+1}} y' \mathrm{d}x = \int_{x_n}^{x_{n+1}} f(x, y)\, \mathrm{d}x.$$

Then

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} f(x, y)\, \mathrm{d}x. \tag{2}$$

The function $f(x, y)$ in (2) is approximated by the Lagrange polynomial which interpolates the set of points mentioned. Evaluating the integral using **MATHEMATICA** will give the formula for the first point in terms of r as follows.

The first point:

$$\begin{aligned} y_{n+1} = y_n + h\Bigg[& \frac{17010r^3 + 13530r^2 + 4005r + 413}{241920r^4(r+1)(2r+1)(4r+1)(4r+3)} f_{n-4} - \frac{6480r^3 + 4920r^2 + 1335r + 118}{6480r^4(r+1)(3r+1)(3r+2)(3r+4)} f_{n-3} \\ & + \frac{34020r^3 + 23370r^2 + 5340r + 413}{20160r^4(r+1)(r+2)(2r+1)(2r+3)} f_{n-2} - \frac{136080r^3 + 63960r^2 + 12015r + 826}{15120r^4(r+1)(r+2)(r+3)(r+4)} f_{n-1} \\ & + \frac{253008r^4 + 141750r^3 + 43050r^2 + 6675r + 413}{725760r^4} f_n + \frac{325584r^4 + 394800r^3 + 190260r^2 + 41010r + 3275}{15120(r+1)(2r+1)(3r+1)(4r+1)} \\ & \times f_{n+1} - \frac{44352r^4 + 43050r^3 + 17220r^2 + 3165r + 220}{20160(r+1)(r+2)(2r+1)(3r+2)} f_{n+2} + \frac{7632r^4 + 7200r^3 + 2820r^2 + 510r + 35}{6480(r+1)(r+3)(2r+3)(4r+3)} f_{n+3} \\ & - \frac{19152r^4 + 17850r^3 + 6930r^2 + 1245r + 85}{241920(r+1)(r+2)(r+4)(3r+4)} f_{n+4} \Bigg]. \end{aligned} \tag{3}$$

The approximate value for the second point, y_{n+2}, is derived by integrating (1) over the interval $[x_{n+1}, x_{n+2}]$. Approximating f using the Lagrange polynomial and lastly evaluating the integral using **MATHEMATICA**, the formula for the second point in terms of r is obtained as follows.

The second point:

$$\begin{aligned} y_{n+2} = y_{n+1} + h\Bigg[& -\frac{6930r^3 + 18810r^2 + 15525r + 3997}{241920r^4(r+1)(2r+1)(4r+1)(4r+3)} f_{n-4} + \frac{2640r^3 + 6840r^2 + 5175r + 1142}{6480r^4(r+1)(3r+1)(3r+2)(3r+4)} f_{n-3} \\ & - \frac{13860r^3 + 32490r^2 + 20700r + 3997}{20160r^4(r+1)(r+2)(2r+1)(2r+3)} f_{n-2} + \frac{55440r^3 + 88920r^2 + 46575r + 7994}{15120r^4(r+1)(r+2)(r+3)(r+4)} f_{n-1} \\ & - \frac{19152r^4 + 57750r^3 + 59850r^2 + 25875r + 3997}{725760r^4} f_n \\ & + \frac{174384r^4 + 478800r^3 + 454860r^2 + 181710r + 26165}{15120(r+1)(2r+1)(3r+1)(4r+1)} f_{n+1} \\ & + \frac{76608r^4 + 261450r^3 + 306180r^2 + 149085r + 25820}{20160(r+1)(r+2)(2r+1)(3r+2)} f_{n+2} \end{aligned}$$