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Oscillation criteria for two-dimensional dynamic systems on time scales

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ABSTRACT

We establish some oscillation criteria for the two-dimensional dynamic system

$$\begin{cases} x^{\Delta}(t) = b(t)g[y^{\sigma}(t)], \\ y^{\Delta}(t) = -a(t)f[x(t)], \end{cases}$$

on a time scale $\mathbb{T}$. Our results not only unify the oscillation of two-dimensional differential systems and difference systems, but include the oscillation results for differential systems and provide new oscillation criteria for difference systems. Several examples are considered to illustrate the main results.

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1. Introduction

In this paper, we investigate the two-dimensional dynamic system

$$\begin{cases} x^{\Delta}(t) = b(t)g[y^{\sigma}(t)], \\ y^{\Delta}(t) = -a(t)f[x(t)], \end{cases} \tag{1.1}$$

on a time scale $\mathbb{T}$. Since we are interested in the oscillatory behavior of the solution of system (1.1) near infinity, we shall assume throughout this paper that the time scale $\mathbb{T}$ is unbounded above. We assume $t_0 \in \mathbb{T}$ and it is convenient to let $t_0 > 0$, and define the time scale interval $[t_0, \infty)_{\mathbb{T}}$ by $[t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$. For system (1.1), we assume that:

(A1) $a \in \mathbf{C}_{rd}([t_0, \infty)_{\mathbb{T}}, \mathbb{R})$, $b \in \mathbf{C}_{rd}([t_0, \infty)_{\mathbb{T}}, [0, \infty))$, and $b(t)$ is not identically zero on $[t_0, \infty)_{\mathbb{T}}$ such that $\int_{t_0}^{\infty} b(t)\Delta t = \infty$;

(A2) $f, g \in \mathbf{C}(\mathbb{R}, \mathbb{R})$ are nondecreasing functions with sign property

$$uf(u) > 0 \text{ and } ug(u) > 0 \text{ for all } u \in \mathbb{R} - \{0\}.$$

Throughout the paper, if

$$\int_{t_0}^{\infty} a(s)\Delta s \quad \text{exists as a real number,}$$

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we denote the function A by

$$A(t) = \int_{\sigma(t)}^{\infty} a(s)\Delta s, \quad t \geq t_0.$$

For simplicity, we also define the function B as follows:

$$B(t) = \int_{t_0}^{t} b(t)\Delta t, \quad t > t_0.$$

By the solution of system (1.1), we mean a pair of nontrivial real-valued functions $(x(t), y(t))$ which has property $x, y \in \mathbf{C}_{rd}^1([t_0, \infty)_{\mathbb{T}}, \mathbb{R})$ and satisfies system (1.1) for $t \in [t_0, \infty)_{\mathbb{T}}$. Our attention is restricted to those solutions $(x(t), y(t))$ of system (1.1) which exist on some half-line $[t_x, \infty)_{\mathbb{T}}$ and satisfy $\sup\{|x(t)| + |y(t)| : t > t_x\} > 0$ for any $t_x \geq t_0$. As usual, a continuous real-valued function defined on an interval $[T_0, \infty)$ is said to be oscillatory if it has arbitrarily large zeros, otherwise it is said to be nonoscillatory. A solution $(x(t), y(t))$ of system (1.1) is called oscillatory if both $x(t)$ and $y(t)$ are oscillatory functions, and otherwise it will be called nonoscillatory. System (1.1) will be called oscillatory if its solutions are oscillatory.

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger in his PhD thesis in 1988 in order to unify continuous and discrete analysis (see [11]). Not only can this theory of the so-called "dynamic equations" unify the theories of differential equations and difference equations, but also extend these classical cases to cases "in between", e.g., to the so-called q-difference equations and can be applied on other different types of time scales. Since Hilger formed the definition of derivatives and integrals on time scales, several authors have expounded on various aspects of the new theory, see the paper in [1] and the references cited therein. A book on the subject of time scales in [5] summarizes and organizes much of time scale calculus. The reader is referred to [5], Chapter 1 for the necessary time scale definitions and notations used throughout this paper.

Dynamic system (1.1) includes two-dimensional linear/nonlinear differential and difference systems, which were deeply investigated in the literature.

When $\mathbb{T} = \mathbb{R}$, we have

$$\sigma(t) = t, \qquad \mu(t) = 0, \qquad f^{\Delta}(t) = f'(t), \qquad \int_a^b f(t)\Delta t = \int_a^b f(t)\mathrm{d}t,$$

and system (1.1) is equivalent to the two-dimensional differential system

$$\begin{cases} x'(t) = b(t)g[y(t)], \\ y'(t) = -a(t)f[x(t)]. \end{cases} \tag{1.2}$$

The oscillatory property of system (1.2) has been studied by many authors, see for example [16,17].

When $\mathbb{T} = \mathbb{N}$ and $\{a_n\}$, $\{b_n\}$ are the nonnegative real sequence, we have

$$\sigma(n) = n + 1, \qquad \mu(n) = 1,$$

$$f^{\Delta}(n) = \Delta f(n) = f_{n+1} - f_n, \qquad \int_a^b f(n)\Delta n = \sum_{k=0}^{b-a} f_{a+k},$$

and system (1.1) becomes the two-dimensional difference system

$$\begin{cases} \Delta x_n = b_n g(y_{n+1}), \\ \Delta y_n = -a_n f(x_n). \end{cases} \tag{1.3}$$

The oscillatory property of system (1.3) has been receiving attention. We refer the reader to the papers [10,13–15].

On the other hand, in the particular case where a is positive on $[t_0, \infty)_{\mathbb{T}}$ and $f(u) = u$, $u \in \mathbb{R}$, the dynamic system (1.1) reduces to the second order nonlinear dynamic equation

$$\left[y^{\Delta}(t)/a(t)\right]^{\Delta} + b(t)g[y^{\sigma}(t)] = 0. \tag{1.4}$$

For $a(t) = 1$ for $t \in [t_0, \infty)_{\mathbb{T}}$, (1.4) becomes

$$y^{\Delta\Delta}(t) + b(t)g[y^{\sigma}(t)] = 0. \tag{1.5}$$

The prototype of Eq. (1.5) is the Emden–Fowler dynamic equation

$$y^{\Delta\Delta}(t) + b(t)|y^{\sigma}(t)|^{\lambda-1}y^{\sigma}(t) = 0, \quad \lambda > 0. \tag{1.6}$$

In recent years there has been much research activity concerning the oscillation and nonoscillation of solutions of dynamic equations (1.4)–(1.6) on time scales. We refer the reader to the recent papers [2–4,6–9,12,18–21] and references cited therein. However, to the best of our knowledge, there are no results dealing with the oscillation of the solutions of two-dimensional dynamic systems on time scales upto now. To develop the qualitative theory of dynamic systems on time scales, in this paper, we shall consider the oscillatory property of system (1.1) and establish some oscillation criteria for system (1.1). Our results not only unify the oscillation of two-dimensional differential systems and difference systems, but include the oscillation results for differential systems and provide new oscillation criteria for difference systems. Finally, several examples are considered to illustrate the main results.

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