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Two-point boundary value problems by the extended Adomian decomposition method

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Abstract

In this paper, we present an efficient numerical algorithm for solving two-point linear and nonlinear boundary value problems, which is based on the Adomian decomposition method (ADM), namely, the extended ADM (EADM). The proposed method is examined by comparing the results with other methods. Numerical results show that the proposed method is much more efficient and accurate than other methods with less computational work.

Keywords: Two-point boundary value problem; Adomain decomposition method; Shooting methods

1. Introduction

In this paper, we study two-point boundary value problems of the form

$$u'' = f(x, u, u'), \quad a < x < b, \tag{1}$$

subject to the boundary conditions

$$u(a) = \alpha, \quad u(b) = \beta, \tag{2}$$

where f is continuous on the set $D = \{(x, u, u') | a \leqslant x \leqslant b, u, u' \in \mathbb{R}\}$.

Two-point boundary value problems have been investigated in many application areas. The most common numerical method for solving these problems is to use shooting methods [6,10]. Although shooting methods have many advantages such as a fast solver and a reduced size of system, it also requires a huge amount of computational work in obtaining accurate approximations especially for nonlinear problems.

The Adomian decomposition method (ADM) has been studied by many scientists [1–3,7–9,12] for solving differential and integral problems in many scientific applications. It decomposes the solution into the series which converges rapidly.

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Each component can be easily determined by using a simple recursive relation. Let us rewrite the model problem (1) in operator form as follows:

$$Lu = Nu + \phi, \tag{3}$$

where L is the second-order derivative operator and N is the nonlinear operator that can be defined by $N = \hat{f}$, where $f(x,u,u') = \hat{f}(x,u,u') + \phi(x)$. Applying the inverse operator L^{-1} to both sides of (3), and using the boundary (or initial) condition, we obtain

$$u = g + L^{-1}\phi + L^{-1}Nu, \tag{4}$$

where g represents the term arising from the given boundary (or initial) condition. The standard ADM defines the solution u by the series

$$u = \lim_{n\to\infty} S_n, \quad S_n = \sum_{i=1}^{n} u_i,$$

where each component u_i can be determined recursively as follows:

$$u_0 = g + L^{-1}\phi, \quad u_{i+1} = L^{-1}(Nu_i), \qquad i \geqslant 0. \tag{5}$$

It is well known [1] that the nonlinear function $N(u)$ is usually represented by the infinite series of polynomials called Adomian polynomials A_n

$$N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n), \tag{6}$$

where Adomian polynomials A_n are defined by

$$A_n(u_0, \ldots, u_n) = \frac{1}{n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n} N\left(\sum_{i=0}^{n} u_i \lambda^i\right)\right]_{\lambda=0}. \tag{7}$$

As seen in (4), ADM is based on finding the solution in operator form by taking a suitable inverse operator L^{-1}. Since the operator L is the second-order differential operator, the inverse operator L^{-1} is either twofold definite or indefinite integral.

Let us consider the inverse operator L^{-1} as the twofold definite integral defined by

$$L^{-1} = \int_a^x \mathrm{d}x' \int_a^{x'} \mathrm{d}x''. \tag{8}$$

This implies

$$L^{-1}Lu = u(x) - u(a) - (x-a)u'(a).$$

Thus, the solution in (3) can be written as

$$u = u(a) + (x-a)u'(a) + L^{-1}\phi + L^{-1}Nu. \tag{9}$$

Applying the standard ADM yields the following recursive scheme:

$$u_0 = u(a) + (x-a)u'(a) + L^{-1}\phi, \quad u_{n+1} = L^{-1}Nu_n, \quad n \geqslant 0.$$

In order to determine all other components u_n, $n \geqslant 1$, the zeroth component u_0 has to be determined. However, $u'(a)$ is not defined by the boundary condition so that the zeroth component cannot be directly determined.

Many authors [2,3,7,12] have proposed modified ADMs to overcome this difficulty. In [7,12], $u'(a)$ is set to be a constant, $u'(a) = c$, and it can be determined such that the nth partial sum $S_n(x, c)$ satisfies the boundary condition at