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A perishable inventory system with retrial demands and a finite population

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ABSTRACT

In this article, we consider a continuous review perishable inventory system with a finite number of homogeneous sources of demands. The maximum storage capacity is S. The life time of each items is assumed to be exponential. The operating policy is (s, S) policy, that is, whenever the inventory level drops to s, an order for $Q(= S - s)$ items is placed. The ordered items are received after a random time which is distributed as exponential. We assume that demands occurring during the stock-out period enter into the orbit. These orbiting demands send out signal to compete for their demand which is distributed as exponential. The joint probability distribution of the inventory level and the number of demands in the orbit are obtained in the steady state case. Various system performance measures are derived and the results are illustrated numerically.

1. Introduction

The analysis of perishable inventory systems has been the theme of many articles due to its potential applications in sectors like food, chemicals, pharmaceuticals, photography and blood bank management. The often quoted review articles [17,18] and the recent review articles [19,12] provide excellent summaries of many of these modelling efforts. Most of these models deal with either the periodic review systems with fixed life times or continuous review systems with instantaneous supply of reorders.

In the case of continuous review perishable inventory models with random life times for the items, most of the models assume instantaneous supply of order [15,16,13]. The assumption of positive lead times further increases the complexity of the analysis of these models and hence there are only a limited number of papers dealing with positive lead times. Moreover they are mostly devoted to the systems with base stock policy [14] or fixed reorder level [10]. In all these models, authors assumed that the demands that occurred during stock-out is either backlogged or lost and the number of sources that generate demands are infinite.

In this paper we relax these assumptions. We assume that the demands that occurred during stock-out enters into the orbit and retry for their demands after a random time. The concept of retrial demands in inventory was introduced in [6] and only few papers [21,20] have appeared in this area. However, considerable interest is shown in the study of Queueing models with retrial customers [8,3–5,11].

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We will focus on the case in which the population under study is finite so each individual customer generates his own flow of primary demands. For the analysis of finite source retrial queue in continuous time, the interested reader is referred to [8,2,9,1,7] references therein.

The rest of the paper is organized as follows. In Section 2, we describe the mathematical model. The steady-state analysis of the model is presented in Section 3 and some key system performance measures are derived in Section 4. In Section 5, we calculate the total expected cost rate and in the final section, the results are illustrated numerically.

2. Model description

Consider an inventory system with a maximum stock of S units and the demands originated from a finite population of sources N. Each source is either free or in the orbit at any time. The demand occurrence times form an output stream which is assumed to be the so called quasirandom output; that is, the probability that any particular source generates a request for demand in any interval $(t, t + \mathrm{d}t)$ is $\alpha \mathrm{d}t + o(\mathrm{d}t)$ as $\mathrm{d}t \to 0$ if the source is idle at time t and zero if the source is in orbit at time t, independently of the behaviour of any other sources. The life time of each item is exponential with rate $\gamma(>0)$. As and when the on-hand inventory level drops to a prefixed level $s(\geq 0)$, an order for $Q(= S - s > s)$ units is placed. The lead time distribution is exponential with parameter $\mu(>0)$. The demands occurring during stock-out periods enter into an orbit. These orbiting demands compete for their demands according to an exponentially distribution with parameter $\theta(>0)$. We consider the constant retrial policy, that is, the probability of a repeated attempt is independent of the number of demands in the orbit. We also assume that the inter demand times between primary demands, lead times, life time of each items and retrial demand times are mutually independent random variables.

Notations:

- $[A]_{i,j}$: element/submatrix at ith row, jth column of the matrix A.
- $\mathbf{0}$: zero vector.
- $\mathbf{I}$: identity matrix.
- $\mathbf{e}^T = (1, 1, \ldots, 1)$.

3. Analysis

Let $L(t)$ denote the inventory level and $X(t)$ denote the number of demands in the orbit at time t. From the assumptions made on the input and output processes, it may be verified that the stochastic process $\{(L(t), X(t)), t \geq 0\}$ with state space $E = \{0, 1, \ldots, S\} \times \{0, 1, \ldots, N\}$ is a Markov process. The transition of the process from the state $(i, k)(L(t) = i, X(t) = k)$ to the state $(j, l)(L(t + \mathrm{d}t) = j, X(t + \mathrm{d}t) = l)$ is denoted by

$$(p((i, k), (j, l))),$$

and can be obtained using the following arguments :

- A transition from state (i, k) to state $(i - 1, k)$ takes place when any one of the 'i' items perishes for which the rate is $i\gamma$ or when a primary demand from any one of the $(N - k)$ sources occurs for which the rate is $(N - k)\alpha$. Hence, the intensity of this transition is $i\gamma + (N - k)\alpha$, where $i = 1, 2, \ldots S$, $k = 0, 1, \ldots, N$.
- A retrial demand takes the state of the process from (i, k) to $(i - 1, k - 1)$ and the intensity of this transition is θ, where $i = 1, 2, \ldots, S$, $k = 1, 2, \ldots, N$.
- When the inventory level is zero, any arriving primary demand enters into the orbit. Thus a transition takes place from $(0, k)$ to $(0, k + 1)$ with intensity of transition $(N - k)\alpha$, $k = 0, 1, \ldots, N - 1$.
- A transition from (i, k) to $(i + Q, k)$, for $i = 0, 1, 2, \ldots, s$; $k = 0, 1, \ldots, N$ will take place with the intensity of transition μ when a replenishment for Q items occurs.
- For other transitions from (i, k) to (j, l), except $(j, l) \neq (i, k)$ the rate is zero.
- To obtain the intensity of passage, $p((i, k), (i, k))$ of state (i, k) we note that the entries in any row of this matrix add to zero. Hence the diagonal entry is equal to the negative of the sum of the other entries in that row. More explicitly

$$p((i, k), (i, k)) = -\sum_{\substack{j \\ (j,l)\neq(i,k)}} \sum_{l} p((i, k), (j, l)).$$

Hence we have