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Influence of forecasting electricity prices in the optimization of complex hydrothermal systems

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ABSTRACT

This paper proposes a new method for addressing the short-term optimal operation of a generation company, fully adapted to represent the characteristics of the new competitive markets. We propose an efficient and highly accurate novel method for next-day price forecasting. We model the functional time series with a linear autoregressive functional model which formulates the relationships between each daily function of prices and the functions of previous days. For the optimization problem (formulated within the framework of nonsmooth analysis using Pontryagin's Maximum Principle), we propose a new method that uses diverse mathematical techniques (the Shooting Method, Euler's Method, the Cyclic Coordinate Descent Method). These techniques are well known for the case of functions, but are adapted here to the case of functionals and are efficiently combined to provide a novel contribution. Finally, the paper presents the results of applying our method to a price-taker company in the Spanish electricity market.

1. Introduction

Short-term hydrothermal scheduling (STHS) is known as one of the most important optimization problems of the power systems. STHS has long been an object of interest in the scientific community. Hundreds (and thousands) of papers have been published, and several techniques have been applied to solve this problem such as: Dynamic Programming, Lagrangian Relaxation Methods, Bender's Decomposition Methods, Heuristic Decomposition Methods, Genetic Algorithms, etc. We refer the readers to the following list of excellent books: [1–7], which include numerous references to the published works.

In traditional centralized markets the aim of STHS is to determine the optimal operation schedule of thermal units and hydro-plants which minimizes the total thermal production cost over a short-term period, taking into account the system-wide (coupling) and unit-wise (local) operating constraints. In the new competitive deregulated electricity market, the objective function of a company can be defined as one of maximizing profits over a period of up to one week.

This paper presents the new short-term problems that one firm faces when preparing its offers for the day-ahead market, taking into account the expected market clearing price. Since next-day price forecasting [8–10] is a crucial aspect, the paper proposes an efficient and highly accurate novel next-day price forecasting method. We model the series using a functional approach that considers hourly prices as observations of daily functions of prices. We model this functional time series with a linear autoregressive functional model which formulates the relationships between each daily function of prices and the functions of previous days.

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For the optimization problem, we propose a new method that employs diverse mathematical techniques which are well known for the case of functions. However, they are adapted here to the case of functionals and are efficiently combined to provide a novel contribution. Our technique makes maximum use of the special characteristics of the new competitive markets, leading to an algorithm that is very easily implemented besides being flexible to modeling. The algorithm is not affected by the size of the problem and is guaranteed to converge.

We shall set out our problem as an optimal control problem (OCP) in continuous time, a Lagrange-type functional, with non-regular Lagrangian and non-holonomic inequality constraints [11]. The problem shall be formulated within the framework of nonsmooth analysis [12,13] using Pontryagin's Maximum Principle (PMP). The associated variational problem will be related to solving a two-point boundary value problem (TPBVP). The construction of the solution can be performed with an adapted version of the shooting method [14,15], in combination with a discretized and adapted version of Euler's method. When the problem involves various hydro-plants, we have developed an algorithm of its numerical resolution prompted by the so-called method of cyclic coordinate descent (CCD) [16–18].

In the present paper, we generalize several previous studies in which we analyzed particular aspects of the technique. In [19], we presented the adapted CCD and in [20] we tested its convergence (under certain conditions). The non-regular nature of the Lagrangian of the OCP was analyzed in [21,22], presenting its application to the study of valve points in [23]. These studies focused on traditional, centralized markets. But the deregulated electricity market is where the technique is found to be even more advantageous. Three previous papers by the authors studied this problem under certain simplifications. In [24], our model of the spot market explicitly represents the price of electricity as a known exogenous variable. In [25], the volatility of the spot market price of electricity is represented by a *stochastic model* using *clustering techniques*. Finally, in [26], we propose a very efficient next-day price forecasting method based on a functional approach, but considering a hydrothermal system with only one hydro-plant. In the present paper, we shall show that the solution of hydrothermal systems with hydropredominance is significantly more complex.

The paper is organized as follows. In Section 2 we present the Hydrothermal Problem. In Section 3, we give some basic definitions and preparatory mathematical results which are necessary for our approach. Sections 4 and 5 present the optimal solution and the description of the algorithm. In Section 6, we expound the bases of the self-regressing functional models and present the results of applying the complete model in the Spanish electricity market. In Section 7, we discuss the results of numerical experiments. Finally, Section 8 summarizes the main conclusions of our research.

2. Statement of the hydrothermal problem

The optimization problem of one company is described in this section. Let us assume that our hydrothermal system accounts for n hydro-plants and m thermal plants: the $(H_n - T_m)$ Problem. Let us assume that the thermal subproblem has previously solved a standard unit commitment (UC) problem. We shall thus address the economic dispatch (ED) problem directly.

Let $\Psi_i(P_i(t)) : D_i \subseteq \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ $(i = 1, \dots, m)$ be the quadratic cost functions of the m thermal plants

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t) \tag{1}$$

where P_i is the power generated, and we consider technical restrictions of the type

$$P_i^{\min} \le P_i(t) \le P_i^{\max}; \quad \forall t \in [0, T]. \tag{2}$$

$[0, T]$ being the optimization interval. In prior studies [27,28], it was proven that the problem with m thermal plants may be reduced to the study of a hydrothermal system made up of one single thermal plant, called the*thermal equivalent*: the $(H_n - T_1)$ Problem. We shall denote as the thermal equivalent of $\{\Psi_i\}_1^m$, the function Ψ with $P(t)$ being the power generated by said thermal equivalent.

Besides, our system accounts for n hydro-plants that have pumping capacity. Let $H_i(t, z_i(t), \dot{z}_i(t)) : \Omega_{H_i} = [0, T] \times \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ $(i = 1, \dots, n)$ be the function of *effective hydraulic contribution* of the ith hydro-plant, and we consider technical restrictions of the type

$$H_i^{\min} \le H_i(t, z_i(t), \dot{z}_i(t)) \le H_i^{\max}; \quad \forall t \in [0, T]. \tag{3}$$

$z_i(t)$ being the volume that is discharged up to the instant t by the ith hydro-plant, and $\dot{z}_i(t)$ the rate of water discharge at the instant t by the ith hydro-plant. If we assume that b_i is the volume of water that must be discharged by the ith hydro-plant during the entire optimization interval $[0, T]$, the following boundary conditions will have to be fulfilled:

$$z_i(0) = 0, \qquad z_i(T) = b_i. \tag{4}$$

We shall denote $b = (b_1, \dots, b_n)$, $z = (z_1, \dots, z_n)$, $\dot{z} = (\dot{z}_1, \dots, \dot{z}_n)$, and $H(t, z(t), \dot{z}(t)) = \sum_{i=1}^n H_i(t, z_i(t), \dot{z}_i(t))$ as the function of effective hydraulic contribution of the set of hydro-plants. From the viewpoint of a power generation company, and within the framework of the new deregulated electricity market, transmission losses are not relevant, and will not be considered.