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Some properties of generalized *K*-centrosymmetric *H*-matrices

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Abstract

Every $n \times n$ generalized K-centrosymmetric matrix A can be reduced into a 2×2 block diagonal matrix (see [Z. Liu, H. Cao, H. Chen, A note on computing matrix–vector products with generalized centrosymmetric (centrohermitian) matrices, Appl. Math. Comput. 169 (2) (2005) 1332–1345]). This block diagonal matrix is called the reduced form of the matrix A. In this paper we further investigate some properties of the reduced form of these matrices and discuss the square roots of these matrices. Finally exploiting these properties, the development of structure-preserving algorithms for certain computations for generalized K-centrosymmetric H-matrices is discussed.

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1. Introduction

A matrix A is said to be (*skew*-)*centrosymmetric* if $A = JAJ$ $(A = -JAJ)$, where J is the exchange matrix with ones on the anti-diagonal (lower left to upper right) and zeros elsewhere. This class of matrices find use, for example, in digital signal processing [3], in the numerical solution of certain differential equations [2], in Markov processes [25] and in various physics and engineering problems [9]. See [19] for some properties of centrosymmetric matrices.

Generalized versions of these matrices have been defined in [2,15,20,23].

Definition 1. A matrix $A \in \mathbb{R}^{n\times n}$ is said to be *generalized K-centrosymmetric* if $A = KAK$, and *generalized K-skew-centrosymmetric* if $A = -KAK$, where $K \in \mathbb{I}^{n\times n}$ can be any permutation matrix which is the product of disjoint transpositions (i.e., $K^2 = I$ and $K = K^{\mathrm{T}}$).

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Mirrorsymmetric matrices are a special subclass of generalized K-centrosymmetric matrices with

$$K = \begin{bmatrix} & & J_k \\ & I_p & \\ J_k & & \end{bmatrix}, \quad n = 2k + p,$$

where I_p is the $p \times p$ identity matrix and J_k is the $k \times k$ exchange matrix. They play a role in the analysis of multiconductor transmission line equations [16].

The blurring matrices arising in image reconstruction [7,17] are also a special subclass of generalized K-centrosymmetric matrices with

$$K = \begin{bmatrix} J_l & & & & \\ & \cdot & & & \\ & & J_l & & \\ & & & \cdot & \\ & & & & J_l \end{bmatrix}. \tag{1.1}$$

Symmetric block Toeplitz matrices form another important subclass of generalized K-centrosymmetric matrices with

$$K = \begin{bmatrix} & & & & I_l \\ & & & \cdot & \\ & & I_l & & \\ & \cdot & & & \\ I_l & & & & \end{bmatrix}.$$

They appear in signal processing, trigonometric moment problems, integral equations and elliptic partial differential equations with boundary conditions, solved by means of finite differences, see for instance [6,11,12,24].

This paper focuses on generalized K-centrosymmetric H-matrices. In next section we review the definitions of H-matrices, and some basic properties of these matrices, as well as a reduced form of generalized K-centrosymmetric matrices. Some properties of the reduced form of generalized K-centrosymmetric H-matrices will be investigated in Section 3 and the square root of a generalized K-centrosymmetric is discussed in Section 4. Finally, exploiting these properties discussed in preceding two sections, we develop effective algorithms for different computational tasks: for constructing an incomplete LU factorization of a generalized K-centrosymmetric H-matrix with positive diagonal entries, for iteratively solving linear systems with a generalized K-centrosymmetric H-matrix as coefficient matrix, and for computing the principal square root of a generalized K-centrosymmetric H-matrix with positive diagonal entries.

2. Preliminaries

In this section we begin with some basic notation frequently used in the sequel (see, e.g., [4]). For definiteness, matrices throughout this paper are assumed to be real, and the matrix K denotes a fixed permutation matrix of order n consisting of the product of disjoint transpositions.

Definition 2. A matrix $A = (a_{ij})$ is called: a *Z-matrix* if $a_{ij} \leqslant 0$ for $i \neq j$; an *M-matrix* if A is a Z-matrix and $A^{-1} \geqslant 0$; an *H-matrix* if its comparison matrix $\langle A \rangle$ is an M-matrix, where $\langle A \rangle = (\alpha_{ij})$ with $\alpha_{ii} = |a_{ii}|$ for $i = j$, $\alpha_{ij} = -|a_{ij}|$ for $i \neq j$.

Definition 3. An $n \times n$ matrix A is called a generalized K-centrosymmetric H-matrix if it is an H-matrix and also generalized K-centrosymmetric.

Generalized K-centrosymmetric H-matrices are of interest in, e.g., image reconstruction [7,17]: the problem of high-resolution image reconstruction usually reduces to solving the following linear system:

$$Ex = \hat{b}, \tag{2.1}$$

where E is the blurring matrix which is a generalized K-centrosymmetric matrix with $K = \mathrm{Bdiag}(J_l, \ldots, J_l)$ as in (1.1). The system in (2.1) is ill-conditioned and susceptible to noise. The common scheme to remedy this is to use the