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Geometric construction of quintic parametric B-splines

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Abstract

The aim of this paper is to present a new class of B-spline-like functions with tension properties. The main feature of these basis functions consists in possessing C^3 or even C^4 continuity and, at the same time, being endowed by shape parameters that can be easily handled. Therefore they constitute a useful tool for the construction of curves satisfying some prescribed shape constraints. The construction is based on a geometric approach which uses parametric curves with piecewise quintic components.
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1. Introduction

In several applications it is required to construct smooth functions or parametric curves, interpolating or approximating a given set of data, and reproducing their salient geometric properties. Classical methods based on piecewise polynomials, often do not produce interpolants or approximants satisfying the required constraints. Thus a great deal of research has focused on the study of new function spaces for building constrained curves, see for instance [5].

The methods proposed so far rarely produce curves with (analytic) smoothness order greater than two, [4], even if curves with at least C^3 continuity are often preferable in some industrial applications as in the design of robot trajectories.

Therefore, our recent studies have concentrated attention on the construction of a B-spline-like basis such that any element of the basis is of class C^r with $r \geq 3$ and possesses tension parameters to control its shape. The first results are presented in [9]. The basis functions have been obtained by a simple geometric construction and have shape parameters with a clear geometric interpretation, which is crucial for their automatic selection. The main tool for the proposed construction is given by the parametric techniques which consist in regarding the basis functions

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as special parametric curves having piecewise quintic components. The resulting basis in [9] is of class C^3 and has minimal (four intervals) support; this last property which entails a good localization, has the drawback of being able to model properly only planar curves. Indeed, by construction any curve obtained as the linear combination of such basis elements, with coefficients in $\mathbb{R}^3$, is of course of class C^3 but has zero torsion at the knots. This problem can be solved by considering basis functions with larger support.

The aim of this paper is to present a new one-parameter family of C^4 B-spline-like functions with 6 intervals support possessing shape parameters: as far as we know function spaces with such a high order of smoothness have never been proposed before in the context of constrained interpolation/approximation. These new functions are achieved by a two-step procedure.

First, we construct a two-parameter family of C^3 B-spline-like functions with 6 intervals support, obtained again by the parametric approach considering planar curves with piecewise quintic components and geometric continuity of order three. The goal is reached by coupling the parametric approach with an extension of the geometric construction presented in [3] where $C^2 \cap FC^3$ piecewise quintic curves were obtained. The resulting basis functions overcome the drawbacks of those in [9] and can be profitably used, in practical applications.

The second step consists in extending our construction to the fourth degree of smoothness by means of the geometric continuity of order four in the parametric setting. This final result is based on the detailed description of geometric B-splines given in [2].

We remark that the basis functions in [4] have a three interval support, are of class C^3, allow construction of space curves with non-necessarily zero torsion (see also [3]), and closely imitate the structure of quintic B-splines with double knots. In particular, any three consecutive intervals have a pair of basis functions associated to it. As a consequence, the dimension of the spanned space doubles the number of the intervals (plus a given number of "boundary conditions"). On the other hand, in some practical problems (mainly interpolation) the data extension matches the number of knots: so it is preferable to deal with spaces of the same dimension of the knot sequence (but for a given number of "boundary conditions"). This is exactly achieved by the basis elements proposed in [9] (in such a sense the support of these basis functions is *minimal*) and by the spaces of functions that we construct in the present paper.

The remainder of this paper is organized into 4 sections. In the next one we review the basic ideas of the parametric techniques showing how analytic continuity can be obtained through geometric continuity. In Section 3, we present the construction of B-spline-like functions of class C^3 and analyse their behavior as the shape parameters tend to limit values, while in Section 4 we extend the geometric construction in order to obtain parametric curves with geometric continuity of order four and therefore functions of class C^4. Section 5 is devoted to some applications and concluding remarks.

To prevent some possible ambiguities, we emphasize that in the paper, parametric curves are addressed from two different points of view. First the B-spline-like basis *functions* are constructed as special *parametric curves*. Then, as a possible application, these functions are used in the context of constrained approximation of parametric space curves (see Section 5).

2. Parametric approach

We construct our basis functions as particular planar parametric curves, according to the so-called parametric techniques, introduced some years ago in the context of tension methods for shape preserving interpolation, see for instance [7,8]. To help in the comprehension of the, sometimes heavy, notation, we mention that throughout the paper bold characters denote vectors, while italic characters and Greek letters are used for scalar quantities.

In the parametric approach, the graph of the function $x \to s(x)$ is seen as the support of a particular planar parametric curve. Let us consider the curve:

$$\mathbf{C}(t) := (X(t), Y(t)), \quad t \in [t_0, t_1]. \tag{1}$$

If we assume that

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