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On the construction of local quadratic spline quasi-interpolants on bounded rectangular domains

C. Dagnino, P. Lamberti*

Department of Mathematics, University of Torino, via C.Alberto, 10 - 10123 Torino, Italy

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Abstract

In this paper local bivariate C^1 spline quasi-interpolants on a criss-cross triangulation of bounded rectangular domains are considered and a computational procedure for their construction is proposed. Numerical and graphical tests are provided.

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1. Introduction

Let $\Omega = [a, b] \times [c, d]$ be a rectangular domain, decomposed into mn subrectangles by the two partitions

$$X_m = \{x_i, 0 \le i \le m\}, \quad Y_n = \{y_j, 0 \le j \le n\},$$

of the segments $[a, b] = [x_0, x_m]$ and $[c, d] = [y_0, y_n]$, respectively. The so called criss-cross triangulation $\mathcal{T}_{mn}$ of Ω is defined by drawing the two diagonals in each subrectangle. We define $S_2^1(\mathcal{T}_{mn}) = \{s \in C^1(\Omega)$: the restriction of s to each triangle is an element of $\mathbb{P}_2\}$, where $\mathbb{P}_\ell$ is the space of polynomials in two variables of total degree less than or equal to ℓ.

Let also $\{B_{ij}, (i, j) \in K_{mn}\}$, with $K_{mn} = \{(i, j) : 0 \le i \le m + 1,\ 0 \le j \le n + 1\}$, be the collection of $(m + 2)(n + 2)$ B-splines [1,4,17], with knots:

$$\begin{aligned} &x_{-2} \le x_{-1} \le a = x_0 < x_1 < \cdots < x_m = b \le x_{m+1} \le x_{m+2}, \\ &y_{-2} \le y_{-1} \le c = y_0 < y_1 < \cdots < y_n = d \le y_{n+1} \le y_{n+2}, \end{aligned} \tag{1}$$

that generate the space $S_2^1(\mathcal{T}_{mn})$. They can be computed both in piecewise polynomial (pp) form, by using the conformality condition method [3], and in Bernstein–Bézier (B–B) form [12].

* Corresponding author.
E-mail addresses: catterina.dagnino@unito.it (C. Dagnino), paola.lamberti@unito.it (P. Lamberti).

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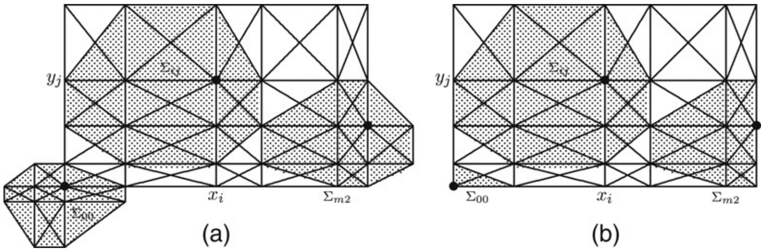

Fig. 1. Some supports of B-splines with (a) simple knots and (b) multiple knots on $\partial\Omega$.

Since $\dim S_2^1(\mathcal{T}_{mn}) = (m+2)(n+2) - 1$ and there is only one linear dependency among the B_{ij}'s, then a basis for $S_2^1(\mathcal{T}_{mn})$ is obtained by deleting any one of them [3].

If in (1) we assume

$$\begin{aligned} &x_{-2} < x_{-1} < a, \quad b < x_{m+1} < x_{m+2}, \\ &y_{-2} < y_{-1} < c, \quad d < y_{n+1} < y_{n+2}, \end{aligned} \tag{2}$$

then we obtain the so called "classical" B-splines with octagonal support Σ_{ij}, simple knots and C^1 smoothness everywhere [3]. We remark that some of their supports are not completely included in Ω, as shown in Fig. 1(a).

If, in (1), we assume

$$\begin{aligned} &x_{-2} \equiv x_{-1} \equiv a, \quad b \equiv x_{m+1} \equiv x_{m+2}, \\ &y_{-2} \equiv y_{-1} \equiv c, \quad d \equiv y_{n+1} \equiv y_{n+2}, \end{aligned} \tag{3}$$

then we have a new set of B-splines B_{ij} [13,15] with multiple knots on the boundary $\partial\Omega$ of Ω and all supports Σ_{ij} included in Ω (Fig. 1(b)). We denote it by $\mathcal{B}_{mn}$. Like the "classical" B-splines, the new ones also satisfy the partition of unity property.

We can distinguish three kinds of "modified" B_{ij}'s belonging to $\mathcal{B}_{mn}$. There are:

(i) a first-boundary-layer of $2m + 2n + 4$ B-splines, with triple knots on $\partial\Omega$, i.e.: $B_{i0},\ B_{i,n+1},\ 0 \le i \le m+1,\ B_{0j},\ B_{m+1,j},\ 1 \le j \le n$;

(ii) a second-boundary-layer of $2m + 2n - 4$ B-splines, with double knots on $\partial\Omega$, i.e.: $B_{i1},\ B_{in},\ 1 \le i \le m,\ B_{1j},\ B_{mj},\ 2 \le j \le n-1$;

(iii) $(m-2)(n-2)$ inner B-splines, with simple knots, i.e.: $B_{ij},\ 2 \le i \le m-1,\ 2 \le j \le n-1$, coinciding with the corresponding "classical" ones.

The knot multiplicity affects the B-spline smoothness, i.e. B_{ij} is $2 - r$ differentiable, if r is the knot multiplicity. Therefore the first-boundary-layer B-splines have a jump on $\partial\Omega$, the second-boundary-layer ones are C^0 on $\partial\Omega$ and the inner ones are C^1 everywhere. Moreover they can be expressed in terms of "classical" B-splines [8].

We recall [6] that the second-boundary-layer B-splines and the inner ones coincide with the so called "interior", "side" and "corner" B-splines, spanning the space of C^1 quadratic piecewise polynomials with boundary conditions [2].

Figs. 2–5 show some B-splines with (a) simple knots and (b) multiple knots on $\partial\Omega$.

This paper deals with discrete local quadratic spline quasi-interpolants (q-i's), defined on a criss-cross triangulation $\mathcal{T}_{mn}$ of Ω and generated by the B-splines belonging to $\mathcal{B}_{mn}$.

In Section 2 we introduce the spline q-i's and we recall some of their properties. In Section 3 we propose a procedure, developed in Matlab, for their generation and we give numerical and graphical test results.

2. Local quadratic C^1 spline quasi-interpolants

We consider linear operators

$$Q : C(\Omega) \to S_2^1(\mathcal{T}_{mn}),$$