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A smoothing Newton-type method for generalized nonlinear complementarity problem ☆

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Abstract

By using a new type of smoothing function, we first reformulate the generalized nonlinear complementarity problem over a polyhedral cone as a smoothing system of equations, and then develop a smoothing Newton-type method for solving it. For the proposed method, we obtain its global convergence under milder conditions, and we further establish its local superlinear (quadratic) convergence rate under the BD-regular assumption. Preliminary numerical experiments are also reported in this paper.

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1. Introduction

Let F, G be continuously differentiable mappings from R^n to R^m, $\mathscr{K}$ be a nonempty closed convex cone in R^m and $\mathscr{K}^\circ$ denote its polar cone. The generalized nonlinear complementarity problem, denoted by GNCP$(F, G, \mathscr{K})$, is to find a vector $x^* \in R^n$ such that

$$F(x^*) \in \mathscr{K}, \quad G(x^*) \in \mathscr{K}^\circ, \quad F(x^*)^\top G(x^*) = 0.$$

This problem has many interesting applications in such as engineering and economics, and is a wide class of problems that contains the classical nonlinear complementarity problem, abbreviated as NCP, as a special case, see, e.g. [1,6,10] and references therein. To solve it, one usually reformulates it as a minimization problem over a simple set or an unconstrained optimization problem, see [17] for the case that $\mathscr{K}$ is a general cone, and see [9,10] for the case that $\mathscr{K} = R^n_+$. The conditions under which a stationary point of the reformulated optimization is a solution of the GNCP$(F, G, \mathscr{K})$ were also provided in the literature.

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In this paper, we consider the GNCP(F, G, $\mathscr{K}$) for the case that $m = n$, and $\mathscr{K}$ is a polyhedral cone in R^n, i.e., there exist $A \in R^{s\times n}$, $B \in R^{t\times n}$ such that

$$\mathscr{K} = \{v \in R^n | Av \geqslant 0,\ \ Bv = 0\}.$$

It is easy to verify that its polar cone $\mathscr{K}^\circ$ assumes the following representation:

$$\mathscr{K}^\circ = \{u \in R^n | u = A^\top\lambda_1 + B^\top\lambda_2, \lambda_1 \geqslant 0, \lambda_1 \in R^s, \lambda_2 \in R^t\}.$$

From now on, the GNCP(F, G, $\mathscr{K}$) is specialized over a polyhedral cone, and in the subsequent analysis we abbreviate it as GNCP for simplicity. In [1], Andreani et al. reformulated the problem as a smooth optimization problem with simple constraints and presented the sufficient conditions under which a stationary point of the optimization problem is a solution of the concerned problem. Later, Wang et al. [18] reformulated the problem as a system of nonlinear and nonsmooth equations, and proposed a nonsmooth Levenberg–Marquardt method for solving it.

It is well known that the smoothing Newton-type method received much attention in solving such as NCP and minimization problem due to its high efficiency [2,3,5,14,16]. It seems reasonable to ask if this kind of method can be applied to the GNCP, and this actually constitutes the main motivation of this paper. In the rest of this paper, we will first present a new reformulation of the GNCP by using a new type of smoothing function, and then develop a smoothing Newton-type method for solving it which guarantees the monotonicity of the generated sequence of the objective function. Under milder conditions, we show that any accumulation point of the generated sequence is a solution of the GNCP, and we also establish the local superlinear (quadratic) convergence rate of the proposed algorithm under the BD-regular assumption. Preliminary numerical experiments show the efficiency of the proposed algorithm.

To end this section, we give some standard notions used in this paper: for a continuously differentiable function $\Gamma : R^n \to R^m$, we denote the Jacobian of Γ at $x \in R^n$ by $\Gamma'(x) \in R^{m\times n}$, whereas the transposed Jacobian is denoted as $\nabla\Gamma(x)$. In particular, if $m = 1$, $\nabla\Gamma(x)$ is a column vector. We use $x^\top y$ to denote the inner product of vectors $x, y \in R^n$, and use $[a]_i$ or a_i to denote the ith component of the vector $a \in R^n$. The null space of a matrix B is denoted by $\mathscr{N}(B)$.

2. Preliminaries

In [18], the authors reformulated the GNCP as a system of nonlinear equations based on the following Fischer function [8]:

$$\phi_{\mathrm{F}}(a, b) = \sqrt{a^2 + b^2} - a - b \quad \text{for } a, b \in R$$

as is seen from the following conclusion.

Lemma 2.1. *$x^* \in R^n$ is a solution of the GNCP if and only if there exist $\lambda_1^* \in R^s$ and $\lambda_2^* \in R^t$, such that*

$$\begin{cases} \Phi_{\mathrm{F}}(AF(x^*), \lambda_1^*) = 0, \\ BF(x^*) = 0, \\ G(x^*) - A^\top\lambda_1^* - B^\top\lambda_2^* = 0, \end{cases}$$

where $\Phi_{\mathrm{F}}(a, b) = (\phi_{\mathrm{F}}(a_1, b_1), \phi_{\mathrm{F}}(a_2, b_2), \ldots, \phi_{\mathrm{F}}(a_s, b_s))^\top$ for $a, b \in R^s$.

Now, we will establish a new type of smoothing reformulation of the GNCP based on the following smoothing approximation function to the Fischer function:

$$\phi(\varepsilon, a, b) = \sqrt{a^2 + b^2 + \alpha\varepsilon^2} - a - b, \quad a, b, \varepsilon \in R,$$

where $\alpha > 0$ is a constant.