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Approximation by interpolating variational splines☆

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Abstract

In this paper we present an approximation problem of parametric curves and surfaces from a Lagrange or Hermite data set. In particular, we study an interpolation problem by minimizing some functional on a Sobolev space that produces the new notion of interpolating variational spline. We carefully establish a convergence result. Some specific cases illustrate the generality of this work.

MSC: 65D05; 65D07; 65D10; 65D17

Keywords: Variational curve; Variational surface; Interpolation; Spline

1. Introduction

In geology, geophysics and other Earth sciences, construction problems of curves and surfaces from a Lagrange or Hermite data set are commonly encountered. Such curves or surfaces verify some physical or geometrical conditions such as preserving shape, some fairness criteria, etc.

In recent years, different techniques for the construction of a curve or surface have been developed, for example, interpolation by spline functions, based on the minimization of a certain functional in a Sobolev space from data such as that mentioned above. In this context, we have extended for the smoothing problem using variational splines in [7], and we present the discrete problem for the numerical method in [8]. An exemplary application of this most recent work is presented in [9].

This work introduces an interpolation problem involving variational splines as an extension of the D^m-splines theory [1]; there are some recent ideas to generalize the classical notion of splines, see, for example, [11,12], but such papers focus different applications with respect to our work. We prove the existence and uniqueness of the solution of such a problem. Then, we establish a convergence result. The minimized functional contains various terms, which are controlled by a parameter vector. Likewise, in order to complete and justify the extension of this work, we prove the generality of our method by giving some new particular cases and other ones seen in the same literature.

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The paper is organized as follows. In Section 2, we briefly recall some preliminary notations and results. In Section 3, we introduce the notion of the interpolating variational spline, and then we prove the convergence result of such a function to the function from whence the data proceed in Section 4. Finally, Section 5 is devoted to illustrating particular cases to reflect the generality of this work.

2. Notations and preliminaries

Let $p, n, m, \mu \in \mathbb{N}^*$ such that

$$m > \frac{p}{2} + \mu. \tag{1}$$

We denote by $\langle\cdot\rangle_{\mathbb{R}^n}$ and $\langle\cdot,\cdot\rangle_{\mathbb{R}^n}$, respectively, the Euclidean norm and inner product in $\mathbb{R}^n$. Let Ω be a non-empty open bounded set of $\mathbb{R}^p$ with a Lipschitz-continuous boundary (see [2]). We will use the usual Sobolev space $H^m(\Omega;\mathbb{R}^n)$ equipped with the inner semi-products, for all $0 \leqslant \ell \leqslant m$,

$$(u,v)_{\ell,\Omega,\mathbb{R}^n} = \sum_{|\beta|=\ell} \int_\Omega \langle \partial^\beta u(x), \partial^\beta v(x)\rangle_{\mathbb{R}^n}\,\mathrm{d}x$$

with $\beta = (\beta_1,\ldots,\beta_p) \in \mathbb{N}^p$ and $|\beta| = \beta_1 + \cdots + \beta_p$. The corresponding semi-norms $|u|_{\ell,\Omega,\mathbb{R}^n} = (u,u)^{1/2}_{\ell,\Omega,\mathbb{R}^n}$ and the norm $\|u\|_{m,\Omega,\mathbb{R}^n} = (\sum_{\ell\leqslant m} |u|^2_{\ell,\Omega,\mathbb{R}^n})^{1/2}$. From Sobolev's imbedding theorem we obtain the continuous injection

$$H^m(\Omega;\mathbb{R}^n) \overset{c}{\subset} C^\mu(\overline{\Omega};\mathbb{R}^n), \tag{2}$$

where $C^\mu(\overline{\Omega};\mathbb{R}^n)$ stands for the space of bounded functions uniformly continuous on Ω, together with all their partial derivatives of order $\leqslant \mu$.

Also, we designate by $\mathbb{R}^{N,n}$ the space of real matrices with N rows and n columns, with the inner product $\langle A,B\rangle_{N,n} = \sum_{i,j=1}^{N,n} a_{ij}b_{ij}$, and the corresponding norm $\langle A\rangle_{N,n} = (\langle A,A\rangle_{N,n})^{1/2}$.

Now, suppose we are given:

- an ordered finite subset A of distinct points of $\overline{\Omega}$;
- an ordered finite set of linear applications Σ of the type

$$\Phi : v \longmapsto \partial^\gamma v(a), \quad |\gamma| \leqslant \mu \quad \text{with } \gamma \in \mathbb{N}^p, \tag{3}$$

  with $a \in A$ such that each point of A is associated with at least one element of Σ;
- an ordered finite set Θ of continuous inner semi-products defined in the Sobolev space $H^m(\Omega;\mathbb{R}^n)$.

Let $N_1 = \operatorname{card}\Sigma$ and $N_2 = \operatorname{card}\Theta$. We consider the linear continuous operator $L : C^\mu(\overline{\Omega};\mathbb{R}^n) \longrightarrow \mathbb{R}^{N_1,n}$ defined by $Lv = (\Phi(v))_{\Phi\in\Sigma}$. Likewise, let $\alpha : H^m(\Omega;\mathbb{R}^n) \times H^m(\Omega;\mathbb{R}^n) \longrightarrow \mathbb{R}^{N_2}$ be the application defined by $\alpha(u,v) = (\Psi(u,v))_{\Psi\in\Theta}$.

We suppose that

$$\operatorname{Ker} L \cap P_{m-1}(\Omega;\mathbb{R}^n) = \{0\}, \tag{4}$$

where $P_{m-1}(\Omega;\mathbb{R}^n)$ designates the space of all polynomials defined on Ω with a total degree of less than $m-1$.

3. Interpolating variational spline

Let β be a data matrix of $\mathbb{R}^{N_1,n}$, and let us consider the set

$$K = \{v \in H^m(\Omega;\mathbb{R}^n) \mid Lv = \beta\}.$$