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Numerical aspects in the SGBEM solution of softening cohesive interface problems

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Abstract

Some numerical aspects of a new symmetric Galerkin boundary integral formulation presented in [A. Salvadori, A symmetric boundary integral formulation for cohesive interface problems, *Comput. Mech.* **32** (2003) 381–391] connected with a like arc-length technique for softening cohesive interface problems introduced in [F. Freddi, Cohesive interface analysis via boundary integral equations, Ph.D. thesis, University of Bologna, Italy, 2004] are here investigated. Further, if the problem is invariant with respect to a finite group $\mathscr{G}$ of congruences of the Euclidean space $\mathbb{R}^2$, we propose to reduce the computational cost of the symmetric Galerkin boundary element method (SGBEM) matrix evaluation and linear system solution using suitable restriction matrices strictly related to a system of irreducible matrix representation of $\mathscr{G}$. Some numerical simulations are presented and discussed.

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Keywords: Boundary integral equation; Cohesive interface; Symmetric Galerkin Boundary Element Method (SGBEM); Arc-length technique; Restriction matrices

1. Introduction

The cohesive forces acting at the interface between bodies are actually one of the most important constitutive parameters, determining the strength and stability of structures. For example, displacement-softening interface responses often imply a global strain-softening structural response. In the calculation process, to follow the quasi-static equilibrium path beyond the onset of snap-through or snap-back, the basic idea for a flexible incremental control technique is that the step is specified by a constraint equation, which involves both the problem unknowns and the load multiplier. Since failure is localized along the interfaces, global constraint equations including all the problem unknowns seem to be redundant to produce a converging solution, essentially because they involve unknowns that are not directly responsible for the equilibrium instabilities [3].

In this paper, some numerical aspects of a new symmetric Galerkin boundary integral formulation introduced in [15] and connected with a like arc-length technique [12,14] for softening cohesive interface problems widely studied in [9],

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are investigated. In particular, we intend to answer to some open problems specified at the end of [15] and to present numerical experiments still lacking, to the authors' knowledge, in literature.

In our opinion, the symmetric integral model proposed in [15] appears to be ideally suited for the analysis of cohesive interface problems. In fact, SGBEM involves only variables on the boundary Γ and relegates all non-linearity to a discontinuity locus, say Γ_c (the cohesive interface). The variables can be condensed by inverting a symmetric matrix of coefficients, while any FEM would involve domain variables as well and in the traditional BEMs the matrix to invert is non-symmetric. Furthermore, the SGBEM exhibits superior accuracy and convergence properties with respect to BEMs, as it is now widely recognized, whereas the significant mathematical and numerical difficulties related to the hypersingular integrations nowadays can be regarded as largely overcome [1,7].

Further, if the problem is invariant with respect to a finite group $\mathscr{G}$ of congruences of the Euclidean space $\mathbb{R}^2$, it was shown in [2] that the use of BIE discretizations, suitably adapted to respect symmetry properties, yields useful decompositions, which can reduce the computational cost, the memory storage and can give advantages in terms of precision of the approximate solution. In this paper we use a technique for exploiting possible symmetry properties of inteface problems, based upon suitable *restriction matrices* (see [2]) strictly related to a system of irreducible matrix representations of $\mathscr{G}$ and to the mesh defined on the boundary of the problem domain. Using these restriction matrices we can decompose the invariant discrete problem into independent subproblems with reduced dimension with respect to the original one; a global solution is obtained from superposition of all partial results.

Hence, at first we will describe some numerical algorithms suitable for an efficient evaluation of the approximate solution of the SGBEM linear system coming from the discretization of a cohesive interface problem; then we will present several numerical results related to some classical problems concerning the behavior of bond between externally bonded fiber reinforced polymers (FRP) and concrete.

2. The problem

This section provides a very brief review of boundary integral equations for softening cohesive interface problems. The reader is asked to consult the cited references and in particular paper [15] for further details. We consider, in a right-hand Cartesian reference system, two-dimensional elastic bodies occupying the domains Ω_1 and Ω_2, which represent the undistorted natural reference configuration of two homogeneous solids, bounded by exterior Lipschitz boundaries Γ^1, Γ^2 with outward unit normal $\mathbf{n}^1$ and $\mathbf{n}^2$, respectively, and connected by a non-linear cohesive interface $\Gamma_c = \Gamma^1 \cap \Gamma^2$. Domain boundaries Γ^i, $i = 1, 2$, are divided into three parts corresponding to boundary conditions and interface: $\Gamma^i = \Gamma^i_D \cup \Gamma^i_N \cup \Gamma^i_c$, where Γ^i_c is the cohesive interface Γ_c conceived as belonging to the boundary Γ^i. According to small displacements and strains theory, we consider in absence of body forces domains response to quasi-static external actions: tractions $\mathbf{p}_i = \sigma(\mathbf{u}_i)\mathbf{n}^i \equiv \bar{\mathbf{p}}_i$, on Γ^i_N (Neumann condition), where σ is the Cauchy stress tensor related to the strain tensor by Hookes law, and displacements $\mathbf{u}_i \equiv \bar{\mathbf{u}}_i$ on Γ^i_D (Dirichlet condition). Small displacements and strains hypothesis implies:

$$\mathbf{n}^1(\mathbf{x}) = -\mathbf{n}^2(\mathbf{x}), \quad \mathbf{x} \in \Gamma_c, \tag{1}$$

and the equilibrium condition reads:

$$\mathbf{p}_1(\mathbf{x}) = -\mathbf{p}_2(\mathbf{x}), \quad \mathbf{x} \in \Gamma_c. \tag{2}$$

Moreover, we consider here the following assumptions: the (known a priori) interface Γ_c is locus of possible displacement discontinuities $\mathbf{w}$, and equilibrate traction act. These discontinuities $\mathbf{w}$ on the interface need to define a reference line that will be identified with the interface Γ_c between domains Ω_1 and Ω_2. Fixing the parameter t in a *time-like* interval $T = [0, t_f]$, two isomorphisms $\mathbf{x} = \mathbf{h}_i(\mathbf{x}_i(t))$, $i = 1, 2$, between the reference line Γ_c and the boundaries Γ^i_c, are set such that we can define the displacements $\mathbf{u}_i$ on Γ_c:

$$\forall \mathbf{x}_i(t) \in \Gamma^i_c \quad \exists! \mathbf{x} = \mathbf{h}_i(\mathbf{x}_i(t)) \in \Gamma_c : \mathbf{u}_i(\mathbf{x}_i(t)) = \mathbf{x}_i(t) - \mathbf{x}, \quad i = 1, 2,$$

with the property: $\mathbf{u}_1(\mathbf{x}_1(0)) = \mathbf{u}_2(\mathbf{x}_2(0)) = 0$. A normal $\mathbf{n}_c(\mathbf{x})$ along Γ_c still must be defined, consistently with the definition of relative opening displacement:

$$\mathbf{w}(\mathbf{x}, t) = \mathbf{u}_1(\mathbf{x}_1(t)) - \mathbf{u}_2(\mathbf{x}_2(t)), \quad \mathbf{x}_i \in \Gamma^i_c, \quad \mathbf{w}(\mathbf{x}, 0) = 0. \tag{3}$$