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A new stopping criterion for linear perturbed asynchronous iterations

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Abstract

A new stopping criterion is proposed for asynchronous linear fixed point methods in finite precision. The case of absolute error is considered. The originality of this stopping criterion relies on the fact that all tests are made within the same macroiteration.

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1. Introduction

This paper deals with parallel asynchronous iterative algorithms. This issue is gaining considerable attention with the advent of peer to peer and grid computing (see [1,14]). The paper concentrates on stopping criterion for linear parallel asynchronous iterations in the case where successive approximations are perturbed by round off errors. This topic is particularly important since for nonlinear problems one has to detect convergence of auxiliary linearized problems. Stopping asynchronous iterations is a difficult topic, either with respect to computer science or to numerical analysis. As opposed to the special situation of parallel synchronous scheme, the development of stopping criteria for asynchronous iterations is particularly nontrivial since there is no global clock nor synchronization points between processors, and processors go at their own pace.

Several approaches for the convergence detection of parallel asynchronous iterations have been proposed in the literature. On what concerns approaches related to pure computer sciences aspects we can quote [2,7,18]. For mixed approaches combining computer sciences and numerical analysis we refer to the following works [5,6,9–11,23]. We have proposed several results in the perturbed case (See [12,21]). The perturbation of parallel asynchronous fixed point methods in finite precision has been studied in [20], see also [24] in the linear case and [22] in the sequential linear case. In [21] some predictive results have been given in relationship with backward and forward errors; in particular,

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stopping criteria have been proposed. In this paper, we present an original stopping criterion with respect to absolute error whereby decision is taken on the base of data collected during the same macroiteration. More precisely, the originality of our study consists in making the convergence test within the same macroiteration which ends up at the given current update p, i.e., starting from the update p, we build a macroiteration going back in the sequence of events which are related to previous updates.

Recall that when the successive approximation method is applied to the computation of the fixed point problem associated with a contractive mapping $\widetilde{T}: u \in R^n \rightarrow \widetilde{T}(u) \in R^n$, with constant of contraction l and fixed point $u^\star$, then stopping the iterations when $\|u^p - \widetilde{T}(u^p)\| = \|u^p - u^{p+1}\| \leqslant \eta$, implies $\|u^p - u^\star\| \leqslant \eta/(1-l)$; namely, such a stopping test is simply based on the implication $\|v - \widetilde{T}(v)\| \leqslant \eta \Rightarrow \|u^\star - v\| \leqslant \eta/(1-l)$.

An analogous estimate can be obtained for asynchronous iterations and then used with respect to various kinds of perturbations issued both from the intrinsic nature of asynchronous iterations and from round-off errors. Namely, our stopping criterion, with respect to round-off errors, will be based on the following extension of the previous estimate

$$\begin{cases} \max\limits_{i=1,\ldots,n} \left(\frac{|y_i - T_i(z^i)|}{e_i} \right) \leqslant \delta, \\ \|z^i - y\|_{\varepsilon,\infty} \leqslant \eta, \quad i = 1, \ldots, n, \end{cases} \tag{1}$$

where $e_i, i = 1, \ldots, n$ are the components of the Perron–Frobenius eigenvectors, $u \rightarrow \|u\|_{\varepsilon,\infty}$ is an appropriate weighted uniform norm (see [13]), y is the iterate vector as will be shown in the sequel and $\{z^i\}$ represents a family of iterate vectors which belong to a macroiteration. In this framework, we can derive a stopping criterion distinct to our previous works (see [12,21,24]).

The estimates in perturbed situations are presented in the following sections which include also our main result concerning a stopping criterion for asynchronous iterations. We need mainly three kinds of informations

- the first one concerns the delays and allows to know what are the iterates included in the macroiteration, this information permits one to have previous computed values,
- the second one is the stabilization of the algorithm with respect to a small distance between these iterates, which then includes naturally the estimate (1), and leads to practical stopping test developed in forthcoming sections,
- the last one is the numerical value of a constant of contraction and also basic data related to round-off errors.

This paper is organized as follows. Section 2 deals with linear asynchronous iterations in the context of nonperturbed case and also in the case of perturbation by round-off errors. A new stopping criterion for perturbed linear asynchronous iterations is proposed in Section 3. Finally, very simple examples are presented in Section 4, in order to illustrate our approach.

2. Mathematical background

2.1. Classical asynchronous iterations

Let n be an integer and consider two matrices $A \in \mathscr{L}(\mathbb{R}^n)$ and $B \in \mathscr{L}(\mathbb{R}^n)$ such that $A = I - B$. In the sequel, $b_{ij}, i, j = 1, \ldots, n$ denote the entries of matrix B and $|B|$ the matrix with entries $|b_{ij}|, i, j = 1, \ldots, n$. Assume that the spectral radius of the matrix $|B|$ satisfies

$$\rho(|B|) < 1. \tag{2}$$

In [15,24] it was shown that, for all real given positive numbers ε, there exist a strictly positive vectors $e \equiv e_\varepsilon$ and a positive scalar $\lambda \equiv \lambda_\varepsilon$, satisfying

$$|B|.e \leqslant \lambda.e \quad \text{where } \lambda \in [\rho(|B|), \rho(|B|) + \varepsilon]. \tag{3}$$

The space $\mathbb{R}^n$ being normed by the Perron–Frobenius weighted uniform norm defined by

$$\|u\|_{\varepsilon,\infty} = \max_{1 \leqslant j \leqslant n} \left(\frac{|u_j|}{e_j} \right) \tag{4}$$