Available online at www.sciencedirect.com
ScienceDirect
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
Journal of Computational and Applied Mathematics 211 (2008) 213–222
www.elsevier.com/locate/cam

Approximation by smoothing variational vector splines for noisy data[☆]

A. Kouibia[a,*], M. Pasadas[b]

[a] *Departamento de Matemática Aplicada, Facultad de Ciencias,Universidad de Granada, 18071 Granada, Spain*
[b] *Faculté Polydisciplinaire de Taza, Morroco*

Received 1 February 2006; received in revised form 13 November 2006

Abstract

This paper addresses the problem of constructing some free-form curves and surfaces from given to different types of data: exact and noisy data. We extend the theory of D^m-splines over a bounded domain for noisy data to the smoothing variational vector splines. Both results of convergence for respectively the exact and noisy data are established, as soon as some estimations of errors are given.

MSC: 65D05; 65D07; 65D10; 65D17

Keywords: Variational curve; Variational surface; Smoothing; Fairness constraint; Noisy data; Spline

1. Introduction

Smoothing splines are well known for providing nice curves and surfaces from exact or noisy data. In Geology and Structural Geology the reconstruction and/or approximation of a curve or surface from a scattered data set is a commonly encountered problem. Generally, there are many approaches using some variational method to solve this kind of problem. One approach involves minimizing a functional on a convex set. In [5,6] the authors deal with specific vectorial splines which interpolate some vectorial data by solving some variational spline problems involving the rotational and the divergence-free interpolants. These functions can be explicitly determined.

Some approximating problems fitting data sets use a routine operation (see, for example, [2,3,10,12]). In some applications, the situation is complicated by the great number of data and also because the data are usually not exact. The problem that arises, using smoothing splines, is the choice of the smoothing parameter, denoted generally in most papers dealing with this kind of problem by ε. However, for the convergence result when the data are exact, the parameter ε should not tend too quickly to infinity, and in particular it might remain bounded (cf. López de Silanes and Arcangéli [10]). When the data are perturbed by a random noise, the situation is radically different. In this case it is necessary for the convergence of the approximation that ε should tend to infinity. The convergence result of the

☆ This work has been supported by the Junta de Andalucía (Research group FQM/191) and by the Ministerio de Educación y Ciencia (Research project MTM2005-01403).
* Corresponding author. Tel.: +34 958 240478; fax: +34 958 248596.
E-mail addresses: kouibia@ugr.es (A. Kouibia), mpasadas@ugr.es (M. Pasadas).

0377-0427/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2006.11.016

smoothing D^m-splines for the given exact or noisy data has been studied by Ragozin [13], in the one dimensional case, and by Cox [4], Utreras [14], López de Silanes and Arcangéli [10], Arcangéli and Ycart [3], in the multidimensional case. These authors gave convergence rates, the convergence in the stochastic case being understood in the quadratic mean sense.

Our main purpose in this paper is to extend the convergence and error estimate results of D^m-spline functions for noisy data [10] for the variational spline functions [7]. Such an extension is not obvious because we introduce a new term on the minimized functional weighted by a parameter vector. We study the approximation of some parametric curves and surfaces by variational spline functions from exact and noisy data. Specially, the purpose of this paper is to study the convergence of smoothing variational splines relative to sets of data perturbed by a random noise.

This paper is organized as follows. In Section 2, we briefly recall some preliminary notations and we present the theory of smoothing variational splines. In Section 3, we introduce the smoothing variational spline function for the noisy data, we describe some results of probabilities and we prove a convergence result for noisy data. Moreover, we give some estimations of the errors.

2. Smoothing variational splines

Let $p, n, m \in \mathbb{N}^*$ and $N \in \mathbb{N}$ such that

$$m > \frac{p}{2}. \tag{1}$$

Let $\langle \cdot \rangle_{\mathbb{R}^n}$ and $\langle \cdot, \cdot \rangle_{\mathbb{R}^n}$ denote, respectively, the Euclidean norm and inner product in $\mathbb{R}^n$.

Likewise, for any non-empty open set Ω in $\mathbb{R}^p$, we denote by $H^m(\Omega; \mathbb{R}^n)$ the usual Sobolev space of (classes of) functions u belonging to $L^2(\Omega; \mathbb{R}^n)$, together with all their partial derivatives $\partial^\beta u$, in the distribution sense, of order $|\beta| \leqslant m$, where $\beta = (\beta_1, \ldots, \beta_p) \in \mathbb{N}^p$ and $|\beta| = \beta_1 + \cdots + \beta_p$. This space is equipped with the norm

$$\|u\|_{m,\Omega,\mathbb{R}^n} = \left(\sum_{|\beta| \leqslant m} \int_\Omega \langle \partial^\beta u(x) \rangle^2_{\mathbb{R}^n} \, \mathrm{d}x \right)^{1/2},$$

the semi-norms

$$|u|_{j,\Omega,\mathbb{R}^n} = \left(\sum_{|\beta| = j} \int_\Omega \langle \partial^\beta u(x) \rangle^2_{\mathbb{R}^n} \, \mathrm{d}x \right)^{1/2}, \quad 0 \leqslant j \leqslant m,$$

and the corresponding inner semi-products

$$(u, v)_{j,\Omega,\mathbb{R}^n} = \sum_{|\beta| = j} \int_\Omega \langle \partial^\beta u(x), \partial^\beta v(x) \rangle_{\mathbb{R}^n} \, \mathrm{d}x, \quad 0 \leqslant j \leqslant m.$$

When Ω is a bounded open subset of $\mathbb{R}^p$ with Lipschitz-continuous boundary (in the Nečas [11] sense), it follows from Sobolev's imbedding theorem that

$$H^m(\Omega; \mathbb{R}^n) \text{ is a subset of } C^0(\overline{\Omega}; \mathbb{R}^n) \text{ with continuous injection,} \tag{2}$$

where $C^0(\overline{\Omega}; \mathbb{R}^n)$ stands for the space of functions with values in $\mathbb{R}^n$ which are bounded and uniformly continuous on Ω. We denote by $\mathbb{R}^{N,n}$ the space of real matrices with N rows and n columns equipped by the inner product

$$\langle T, B \rangle_{N,n} = \sum_{i,j=1}^{N,n} t_{ij} b_{ij},$$

and the corresponding norm

$$\langle T \rangle_{N,n} = (\langle T, T \rangle_{N,n})^{1/2}.$$