Available online at www.sciencedirect.com
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Journal of Computational and Applied Mathematics 216 (2008) 14–19
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
www.elsevier.com/locate/cam

Constrained degree reduction of polynomials in Bernstein–Bézier form over simplex domain

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Received 16 December 2006

Abstract

In this paper we show that the orthogonal complement of a subspace in the polynomial space of degree n over d-dimensional simplex domain with respect to the L_2-inner product and the weighted Euclidean inner product of BB (Bézier–Bernstein) coefficients are equal. Using it we also prove that the best constrained degree reduction of polynomials over the simplex domain in BB form equals the best approximation of weighted Euclidean norm of coefficients of given polynomial in BB form from the coefficients of polynomials of lower degree in BB form.

Keywords: Simplex domain; Bernstein polynomial; Bézier curve; Weights; Constrained degree reduction

1. Introduction

Degree reduction of Bézier curves is one of the important problems in CAGD (computer aided geometric design) or the approximation theory. In general, degree reduction cannot be done exactly, so that it invokes approximation problems. Thus many papers dealing with the degree reduction have been published in the recent 30 years. They are classified by different norm in which the distance between polynomials is measured, e.g., in L_∞-norm [8,14], in L_2-norm [16,17,20], in L_1-norm [13] or in L_p-norm [6,12], etc. Furthermore, the constrained degree reduction of Bézier curves with C^{a-1}-constraint at boundary points is developed in many previous literature [1,2,5,7,9,11,15, 18–20,22–25].

Especially, Lutterkort et al. [20] showed that the best degree reduction of polynomial f of degree n over the triangular domain in L_2-norm is equivalent to the best approximation of the vector of BB coefficients of f from all vector of BB coefficients of degree elevated polynomials of degree less than n in the Euclidean norm of the vector. Furthermore, Peters et al. [21] extended the result to polynomials over d-dimensional simplex domain. Ahn et al. [4] extended the result of Lutterkort et al. to the case of constrained degree reduction, and Ahn [3] extended the result of Peters et al. to the case of the constrained degree reduction over triangular domain. In this paper we extend the results above to the case of constrained degree reduction over the d-dimensional simplex domain. We propose an weight which is an extension

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[1] This study was supported by research fundsfrom Chosun University, 2007.

0377-0427/$ - see front matter
doi:10.1016/j.cam.2007.04.001

of the weights in Ahn et al. [4] and in Ahn [3]. We expect that the weight proposed in this paper plays an important role in Bernstein–Bézier representation of the constrained Legendre polynomials [10] over the d-dimensional simplex domain.

In Section 2, we first show that the orthogonal complement of a subspace in the constrained polynomial space of degree n over the simplex domain with respect to L_2-inner product and the weighted Euclidean inner product of BB coefficients are equal for some weights. In Section 3, using the fact we also prove that the best constrained degree reduction of f of degree n over the simplex domain in L_2-norm is equal to the best approximation of the vector of coefficients from all vectors of coefficients of degree elevated polynomials with the constraint in weighted Euclidean norm of vectors.

2. Equivalence of orthogonal complements

Let $\mathbb{P}_n$ be the linear space of polynomials of degree less than or equal to n. It is convenient to introduce the compact notation $\alpha = (\alpha_1, \ldots, \alpha_d)$ to denote d-dimensional multiple of nonnegative integers, and we write $|\alpha| = \alpha_1 + \cdots + \alpha_d$. The Bernstein basis of degree n over the d-dimensional simplex domain Δ is denoted by

$$B^n_{\boldsymbol{\alpha}}(x) = \binom{n}{\alpha} x^\alpha \left(1 - \sum_{i=1}^{d} x_i\right)^{n-|\alpha|}, \quad |\alpha| \leqslant n,$$

where $x = (x_1, \ldots, x_d) \in \Delta$ and $x^\alpha = x_1^{\alpha_1} \cdots x_d^{\alpha_d}$. Thus $\mathbb{P}_n$ has exactly $(n + d)!/d!n!$ basis functions. We collect the basis functions in simplicial arrays of size n

$$B^n := [B^n_\alpha]_{|\alpha| \leqslant n}$$

and with $b = [b_\alpha]_{|\alpha| \leqslant n}$ a simplicial array of reals we write polynomials in BB form as

$$B^n b = \sum_{|\alpha| \leqslant n} B^n_\alpha b_\alpha.$$

For the nonnegative integer $a \leqslant n/(d + 1)$, $\mathbb{P}^a_m$ is denoted by the linear space of the constrained polynomials of degree less than or equal to m over the simplex domain Δ as follows:

$$\mathbb{P}^a_m = \{B^n b \in \mathbb{P}_m : b_\alpha = 0 \text{ for } \alpha \in J^a_n\},$$

where I^a_n and J^a_n are the sets of d-dimensional multi-index α such that

$$I^a_n = \{|\alpha| \leqslant n : \alpha_1 \geqslant a, \ldots, \alpha_d \geqslant a, |\alpha| \leqslant n - a\},$$
$$J^a_n = \{|\alpha| \leqslant n : \alpha \notin I^a_n\}.$$

Then $\mathbb{P}^a_m$ has exactly $(k + d)!/k!d!$ basis functions, where $k = n - (d + 1)a$. We also have

$$f \in \mathbb{P}^a_m \quad \Leftrightarrow \quad f(x) = x^a \left(1 - \sum_{i=1}^{d} x_i\right)^a g(x)$$

for some $g \in \mathbb{P}_k$. For $m < n$ and $a \leqslant m/(d + 1)$, let

$$\mathbb{Q}^a_m = \{f(x) \in \mathbb{P}_m : f(x) = 0 \text{ for } x_1, \ldots, x_n, n - |x| = 0, \ldots, a - 1\},$$

which was also introduced for one or two variables by Ahn et al. [4,3]. Note that $\mathbb{P}_m = \mathbb{P}^0_m = \mathbb{Q}^0_m$. We consider the Lagrange polynomials characterized by

$$Q^n_\alpha(\beta) = \delta_{\alpha,\beta}, \quad |\alpha|, |\beta| \leqslant n.$$

Peters and Reif [21] was already introduced the notation of the Lagrange polynomials Q^n_α. We collect the basis functions in simplicial arrays of size n

$$Q^n := [Q^n_\alpha]_{|\alpha| \leqslant n}$$