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A Newton-type method for constrained least-squares data-fitting with easy-to-control rational curves

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Abstract

While the mathematics of constrained least-squares data-fitting is neat and clear, implementing a rapid and fully automatic fitter that is able to generate a fair curve approximating the shape described by an ordered sequence of distinct data subject to certain interpolation requirements, is far more difficult.

The novel idea presented in this paper allows us to solve this problem with efficient performance by exploiting a class of very flexible and easy-to-control piecewise rational Hermite interpolants that make it possible to identify the desired solution with only a few computations. The key step of the fitting procedure is represented by a fast Newton-type algorithm which enables us to automatically compute the weights required by each rational piece to model the shape that best fits the given data. Numerical examples illustrating the effectiveness and efficiency of the new method are presented.

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1. Introduction

In several applications of computer-aided geometric design we frequently have to deal with 3D point reconstruction problems where a large amount of data is given in the form of an ordered sequence of distinct points describing a target shape in space. Most of these usually contain measurement errors, while only a few are rigorously generated and then turn out to be crucial to the final reconstruction.

As regards the computation of a surface/surface intersection curve, for example, the set of discrete data consists of highly-accurate points (such as border points, turning points and cusp points) precisely detected on the intersection curve, and a sequence of marching points generated from each of the above points by going a step in the direction defined by the local differential geometry of the curve. The first ones, commonly called starting points, will be therefore exactly interpolated, while all the others will be approximated in order to reduce to a minimum the sum of the squares of their distances from the desired curve.

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An analogous situation occurs in the point-based construction of a Gordon-type surface. This time the input data is given by a set of 3D points, describing the complete cross-sections of the target surface, which have to be accurately approximated to produce a fair curve network to be assumed as the surface skeleton. Consistency demands that the cross-section curves agree in value where an x-section crosses a y-section. This means that sequences of points along two transversal directions must possess common intersections and these have to be assumed as positional constraints for the fitting problem.

In both the outlined circumstances – as well as in many other applications – it is therefore desirable to use a fitting method that, on one hand, is able to capture the shape of the overall input data and, on the other, to satisfy the assigned point constraints precisely.

Since, when approximating shapes with a complicated behaviour, it always turns out to be convenient to construct the fitting curve segment-wise by means of a piecewise model defined in the space of conventional polynomial splines or NURBS, our idea consists of using the point constraints (identified by the specific application we are considering, or detected from the curvature and torsion information of the input dataset) to partition the given data into adjacent subsets that can be approximated separately by a curve segment taken from some specified class of appropriate curves.

Taking into account that when the points lie on the intersection of two analytic surfaces, derivative constraints of any order can be easily computed by their parametric representation and, when generating Gordon-type surfaces, additional information like first derivatives and/or higher order derivatives to be assumed by the curve network in the significant locations are also generally available, the most natural solution to this kind of constrained fitting problem can be obtained by using a fitter that implements a piecewise Hermite interpolant. This solution also allows us to greatly simplify the computational process that a standard least-squares minimization problem with associated positional constraints would have required in order to ensure a sufficiently high order of continuity at segment boundaries.

While for pure interpolation there is probably little reason to use a rational form, for approximation purposes allowing weights to be arbitrary makes it possible to produce fitting curves with higher accuracy and fewer control points.

The novel solution we are going to propose will rely therefore on a class of piecewise rational Hermite interpolants. In particular we will adopt here the one introduced in [1,2] because of its flexibility and ease of control. This can also be represented in the conventional NURBS form by assuming multiple knots in the correspondence of the location of the interpolation constraints and letting the control points be dependent both on them and on the weights of the rational representation. In this way, once a procedure for computing the optimal weight values has been designed, the control points turn out to be automatically defined and hence the best-fitting curve results completely determined.

Therefore, differently from standard NURBS fitting procedures, which require a complicated and expensive iterative algorithm to minimize (with respect to knots, control points and weights) a sum of squared Euclidean norms measuring the distance between the point set and the curve to be generated [3,4,9,11–15], the least-squares fitting method we are going to propose will be performed exclusively to identify the choice of weights that guarantees the best reconstruction of the original data. Moreover, while the output of existing algorithms cannot always guarantee a fitting curve with a fair shape (namely with a curvature plot consisting of only a small number of monotone pieces), due to the definition of the novel fitter this follows easily and, whenever the degree of the curve primitive is larger than three, a curvature-continuous approximation of the original data is also ensured.

The organization of the paper is as follows. In Section 2 we introduce the rational Hermite basis to be used as a novel curve primitive for determining the solution of the constrained least-squares problem. Next in Section 3 we develop a strategy for carrying out the automatic computation of the optimal weights to be embodied in the desired rational form, and we describe the overall fitting process in all its steps. Finally, in Section 4 we close the paper by showing some numerical examples that confirm the improved performance of the innovative procedure relative to conventional and reliable approaches (like the well-known `lsqcurvefit` algorithm that is currently implemented in MATLAB's Optimization Toolbox).

2. Least-squares fitting with a novel curve primitive

Given a set of 3D distinct points representing a target shape in space, we seek a NURBS curve that lies close to the assigned data and passes through only a few of them. Let $\{\mathbf{Q}_k\}_{k=0,\dots,M-1}$ denote the given set of points in $\mathbb{R}^3$ and $\Im \subset \{0, \dots, M-1\}$ the subset of $N+1$ ($N \ll M$) indexes specifying all the points $\mathbf{Q}_k$ that the fitting curve must