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Structural assessment under uncertain parameters via interval analysis

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Abstract

An efficient health monitoring system for damage detection in civil engineering structures using on-line monitoring data is being developed to identify any possible damage in short time. The present work is based on the treatment of uncertainties, which is one of the basic common difficulties faced when modelling structures. A methodology, based on interval analysis (IA) theory [R.E. Moore, Interval Analysis, Prentice-Hall, Englewood Cliffs, NJ, 1966] applied to a numerical constraint satisfaction problem (CSP) [J.R. Casas, J.C. Matos, J.A. Figueiras, J. Vehí, O. García, P. Herrero, Bridge monitoring and assessment under uncertainty via interval analysis, in: Ninth International Conference On Structural Safety And Reliability—ICOSSAR2005, 2005. pp. 487–494], is implemented in the damage detection [J.R. Casas, J.C. Matos, J.A. Figueiras, J. Vehí, O. García, P. Herrero, Bridge monitoring and assessment under uncertainty via interval analysis, in: Ninth International Conference On Structural Safety And Reliability—ICOSSAR2005, 2005. pp. 487–494] and modelling system of a long-term monitoring project in order to achieve such an objective. An algorithm is being developed for using such methodology with the obtained data.

Such methodology has been first checked in the laboratory with a simple reinforced concrete structure (loaded up to failure). The obtained results are useful for identifying the load where the structure presents changes in its behaviour. The majority of the structures present a linear elastic behaviour throughout their life. However, they tend to deteriorate; such degradation reflects on results obtained from the long term monitoring system. Structural assessment was successfully performed in this case, enabling its application to real structures.

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1. Introduction

Development of structural health monitoring systems (HMS) has been a subject of increasing activity in recent years. One of the main problems in the structural assessment is the treatment of uncertainty mainly present in numerical models,

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physical and geometrical parameters such as loading, Young modulus, inertia, and so on, and in measured variables such as displacements, strains and rotations. One of the main issues while considering the various sources of uncertainty is how to define objective and reliable criteria for distinguishing between an abnormal behaviour (differences between measured values and those predicted by the model) due to the presence of damages, and the differences between measured and calculated results because of the uncertainty and randomness present in the experimental data, models and physical parameters. When performing long term structural assessment, methodologies that take into account these uncertainties should be implemented in an efficient, fast and user friendly way. In this paper a methodology that considers uncertainty both in the model and in recorded data is presented.

2. Treatment of uncertainty

There exist different techniques for consideration of uncertainties in numerical models and measured variables. In the present paper we focus on the use of the interval analysis (IA). A brief summary of this subject is presented and their application to the management of uncertainties in structural assessment.

2.1. Interval analysis

The initial idea of IA [9,10] is to enclose real numbers in intervals and real vectors in boxes as a method of considering the imprecision of representing real numbers by finite digits in numerical computers. The variables are not deterministic, but take any value between the lower and the upper limits of an interval. The variables are represented by a uniform variation and the probability distribution function is not necessary. IA has become a fundamental nonlinear numerical tool for representing uncertainties or errors, proving properties of sets, solving sets of equations or inequalities and optimizing globally via interval arithmetic [5,7]. A classic interval number is a closed set that includes the possible range of an unknown number. Thus, instead of considering a fixed value a, the following representation is adopted:

$$A' = [\underline{a}, \overline{a}] := \{x \varepsilon \mathbb{R} | \underline{a} \leqslant x \leqslant \overline{a}\}, \tag{1}$$

where $\underline{a}$ is the infimum and $\overline{a}$ the supremum of the interval. The four elementary arithmetic operations $(+, -, \times, \div)$ are extended to intervals. If op denotes an arithmetic operation for real numbers, the corresponding interval arithmetic operation is

$$C = A \operatorname{op} B = \{a \operatorname{op} b | a \varepsilon A,\ \ b \varepsilon B\}. \tag{2}$$

2.1.1. Modal intervals

Modal interval analysis (MIA) is a natural extension of classical interval analysis, where the concept of interval is widened by the set of predicates that are fulfilled by the real numbers [4,12].

Physical intervals have two modalities for practical problems: there exists a value in $[a, b]'(a \leqslant b)$, which satisfies a predicate or some predicates concerned, and for all values in $[a, b]'(a \leqslant b)$, they satisfy a predicate or some predicates concerned. Classical interval analysis cannot distinguish these two types of physical intervals and denotes them as $[a, b]'(a \leqslant b)$ uniformly. MIA does distinguish these two types of physical intervals by denoting them differently, i.e., $[a, b](a \leqslant b)$ for those proper intervals that only require the existence of a value in the domain of $a \leqslant x \leqslant b$ to satisfy a predicate or some predicates concerned and $[b, a](a \leqslant b)$ for those improper intervals that require all the values in the domain of $a \leqslant x \leqslant b$ to satisfy a predicate or some predicates concerned, with $[a, b]$ and $[b, a]$ being denoted as modal intervals henceforth on. The concept of modal intervals is similar to the concept of objects in C + + since it contains not only a purely numerical interval, but also a physical modality of the numerical interval.

A modal interval X is defined as a couple $X = (X', \forall)$ or $X = (X', \exists)$, where X' is its classical interval domain and the quantifiers $\forall$ (universal) and $\exists$ (existential) are a modality selection. Modal intervals of type $X = (X', \exists)$ are defined as *proper intervals*, while intervals of type $X = (X', \forall)$ are designated by *improper intervals*. A modal interval can be represented using its canonical coordinates in the form

$$X = [a, b] = \begin{cases} ([a, b]', \exists) & \text{if } a \leqslant b, \\ ([b, a]', \forall) & \text{if } a \geqslant b. \end{cases} \tag{3}$$