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Temperature sensor based on dielectric optical microresonator

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ABSTRACT

An optical temperature sensor has been presented based on Whispering Gallery Mode (WGM) dielectric microresonator. The effect of Transverse Electric (TE) wave propagation in dielectric micro-spheres presented has been for optical resonances based on WGM. TE waves are characterized both theoretically and experimentally for large size parameter of the micro-spheres. A theoretical model has been developed based on asymptotic approach. The theoretical development is mathematically robust and significantly less complicated than existing approaches presented in the literature. The quality factor of experimental resonance spectra observed in the laboratory is calculated approximately in the order of 10^4 which is sensitive enough to detect micro or nano level temperature changes in the surrounding medium. The sensitivity of the Morphology Dependent Resonance (MDR) temperature sensor is wavelength change of 10^{-9} m for one degree centigrade change in temperature. This sensor could potentially be used for nano technology, Micro-Electro-Mechanical Systems (MEMS) devices, and biomedical applications.

1. Introduction

Nowadays optical technology has been using as a one of the most popular technology in research communities. Several optical techniques have been reported [1,2] in recent years. Among the reported techniques, WGM resonators have earned especial attention due to its unique characteristics [3]. Over the years, dielectric microsphere resonators have been reported for various potential applications which includes coupling of optical power between microcavities and waveguides [4], bio-specific interaction analysis [5], optical acceleration sensors [6], accelerating charged particles [7], thermo-optical switches [8], entanglement of atom [9] are the few that revisited here. Recently, WGM resonators have gained increasing interest to numerical study, such as, analyzing the gap effect between sphere and waveguide [10], simulation of energy transfer from waveguide to microcavities [11], and analyzing the efficient optical coupling and transport phenomena in dielectric microspheres arranged in chains [12].

In recent years, theoretical approaches on MDR, also known as WGM, have reported in the literatures [1,13–17]. However, mathematical approaches reported in the literatures using quantum theory are comparatively complicated. It is observed that WGM resonances and MDR are presented in the literatures based on classical quantum mechanics have some drawbacks such as mathematical formulations are not sufficient enough to explain the MDR peaks, theoretical development cannot explain with experimental results, theories are not well explained, hence difficult to follow, mathematical and theoretical approaches do not give simple usable results. Therefore, reported approaches are less attractive in micro optical sensing.

A complete asymptotic solution of TE wave in dielectric microspheres for large size parameter has been presented which is simpler and mathematically robust than existing approaches presented in the literatures [1,13–17]. The developed theory has compared with the experimental results and it is shown that the present approach is very accurate for large size parameters [18]. The quality factor of experimental resonance spectra observed in the laboratory is calculated approximately in the order of 10^4 which is sensitive enough to detect micro or nano level temperature changes in the surrounding medium. The developed mathematical formulation is used to design WGM temperature sensor. The sensitivity of the MDR temperature sensor is wavelength change of 10^{-9} m for one degree centigrade change in temperature. It is also found that even with varying surrounding temperature, for consecutive MDR peaks, size parameter is equal to the product of consecutive integers and a constant which follow the developed theoretical model presented here.

2. Theoretical formulation

In recent years, fiber-sphere coupler is being used as one of the most popular optical setup for sensing sub-micron or nano level changes in surrounding medium. If a dielectric microsphere coupled with an optical fiber, light wave also known as Electro-Magnetic (EM) field traveling through the optical fiber will tunnel from fiber to microsphere. The light particle also known as photon travels inner surface of the sphere due to total internal reflection when the refractive index of the dielectric microsphere is greater

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Nomenclature

b	arbitrary constant
c	arbitrary constant
E	electric field vector
H	magnetic field vector
h_n^1	spherical Bessel function of third kind
I	integer
j_n	spherical Bessel function of first kind
k	wave number
l	integer
m	relative refractive index
M	vector spherical harmonics
N	vector spherical harmonics
N	integer
n	integer
P_n	legendry polynomials
R	radius of the microsphere (m)
r	radial polar coordinate
x	size parameter
z_n	spherical Bessel functions
θ	zenith polar coordinate
φ	azimuth polar coordinate
λ	wavelength of light (m)
μ	permeability
ε	complex permittivity
ω	angular frequency
ψ	given scalar function

Subscripts

0	free space
1	inside sphere
2	outside sphere
e	even
n	nth instant
o	odd
in	inside the sphere
out	outside the sphere

than the refractive index of surrounding medium. While EM wave travels inside the sphere, waves interfere each other in particular wavelength due to circular travel path of the sphere and produce MDR. Qualitatively, this is described as follows: after circumnavigating the microsphere, the light wave returns to the starting point in phase to interfere constructively with itself and WGM resonance occurs. This constructive interference gives resonance peaks at certain discrete wavelengths and these resonance peaks are known as MDR [14]. In this paper a simple approach has been developed to explain MDR peaks with the variation in surrounding temperatures.

It is considered that there is no incident plane wave and assuming EM wave modes are existed inside the sphere and excited by the evanescent modes from the fiber optic waveguide. The solutions inside the sphere are matched to the scattered field external to the sphere at the boundaries, where it is expected that the externally scattered field decays rapidly in the radial direction.

Electromagnetic Magnetic field in a linear, isotropic, homogeneous medium will satisfy the vector wave equation [19],

$$\nabla^2 \vec{E} + \omega^2 \varepsilon\mu \vec{E} = 0 \tag{1a}$$

$$\nabla^2 \vec{H} + \omega^2 \varepsilon\mu \vec{H} = 0 \tag{1b}$$

The scalar wave equation in spherical polar coordinates can be expressed for a given scalar function ψ, as shown in Fig. 1, is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2\psi}{\partial\phi^2} + \omega^2\varepsilon\mu\psi = 0 \tag{2}$$

Particular solutions and generating functions to Eq. (2) can be expressed as [19]

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi) \tag{3}$$

After substituting Eq. (3) into Eq. (2), three separated equations will yield,

$$\frac{d^2\Phi}{d\phi^2} + g^2\Phi = 0 \tag{4}$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + \left[n(n+1) - \frac{g^2}{\sin^2\theta}\right]\Theta = 0 \tag{5}$$

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + [\omega^2\varepsilon\mu r^2 - n(n+1)]R = 0 \tag{6}$$

The generating functions which satisfy Eq. (2)

$$\psi_{en} = \cos(l\phi)P_n(\cos\theta)z_n(kr) \tag{7a}$$

$$\psi_{on} = \sin(l\phi)P_n(\cos\theta)z_n(kr) \tag{7b}$$

The vector spherical harmonics generated by Eq. (7) are [19]

$$\vec{M}_{en} = \nabla \times (\hat{r}\,\psi_{en}) \quad \text{and} \quad \vec{M}_{on} = \nabla \times (\hat{r}\,\psi_{on}) \tag{8a}$$

$$\vec{N}_{en} = \frac{\nabla \times \vec{M}_{en}}{k} \quad \text{and} \quad \vec{N}_{on} = \frac{\nabla \times \vec{M}_{on}}{k} \tag{8b}$$

Assuming, $\vec{E}$ tangential to sphere surface (TE mode), EM field inside the sphere can be expressed as

$$\vec{E}_{in} = \sum_{n=1}^{\infty} E_n c_n \vec{M}_{on}^{(1)} \tag{9a}$$

$$\vec{H}_{in} = \frac{k_{in}}{\omega\mu}\sum_{n=1}^{\infty} E_n i c_n \vec{N}_{in}^{(1)} \tag{9b}$$

The scattered EM field from the sphere can be expressed as

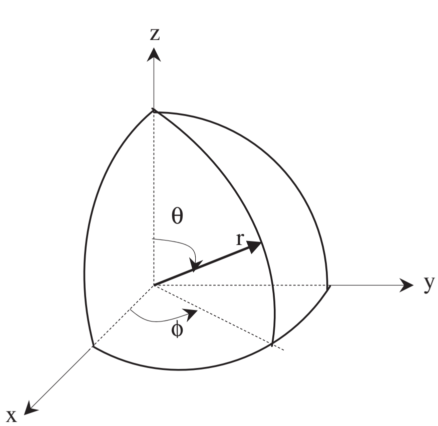

Fig. 1. Spherical polar coordinate, sectional view.