

Finite elements using absolute nodal coordinates for large-deformation flexible multibody dynamics

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Abstract

A family of structural finite elements using a modern absolute nodal coordinate formulation (ANCF) is discussed in the paper with many applications. This approach has been initiated in 1996 by A. Shabana. It introduces large displacements of 2D/3D finite elements relative to the global reference frame without using any local frame. The elements employ finite slopes as nodal variables and can be considered as generalizations of ordinary finite elements that use infinitesimal slopes. In contrast to other large deformation formulations, the equations of motion contain constant mass matrices and generalized gravity forces as well as zero centrifugal and Coriolis inertia forces. The only nonlinear term is a vector of elastic forces. This approach allows applying known abstractions of real elastic bodies: Euler–Bernoulli beams, Timoshenko beams and more general models as well as Kirchhoff and Mindlin plate theories.

Shabana et al. proposed a sub-family of *thick* beam and plate finite elements with large deformations and employ the 3D theory of continuum mechanics. Despite the universality of such approach it has to use extra degrees of freedom when simulating *thin* beams and plates, which case is most important.

In our research, we propose another sub-family of thin beams as well as rectangular and triangle plates. We use Kirchhoff plate theory with nonlinear strain–displacement relationships to obtain elastic forces.

A number of static and dynamic simulation examples of problems with 2D/3D very elastic beams and plate underwent large displacements and/or deformations will be shown in the presentation.

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1. Introduction

The development of flexible multibody dynamics started from problems with small deformations and no rigid-body motion. In the last decades, many efforts have been employed for problems having both large overall motion and large deformations as well as with coupling of rigid and flexible bodies into united structures. Analysis of complex systems becomes impossible without using powerful computer-aided numerical methods. One can find a wide review of various approaches in the paper [10]. Here is a short overview of these methods.

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The *floating reference frame* formulation allows taking into account an arbitrary motion of the body-fixed reference frame. It is a very popular approach now due to its straightforward nature: it just employs rigid-body degrees of freedom (d.o.f.) (usually six) in addition to nodal variables used in structural mechanics. As a consequence, it utilizes the same stiffness matrix to obtain elastic forces. However, a mass matrix, centrifugal and Coriolis inertia forces and even generalized gravity forces appear highly nonlinear in this approach. Moreover, local displacements of the flexible-body continuum are still supposed to be small. This does not allow simulating large deformation problems.

The *incremental finite element* approach is free from this disadvantage but, unfortunately, introduces another one. Because of the use of infinitesimal rotation angles as nodal variables, it leads to linearized kinematic equations and, as a consequence, to inexact description of rigid-body displacements. To avoid this, one can apply an additional concept of *intermediate coordinate system*.

The *large rotation vector* formulation has also been proposed for large deformation problems. It employs finite rotation angles instead of infinitesimal ones and that is why it can correctly represent an arbitrary rigid-body motion.

All these approaches suffer from the high non-linearity of the terms of equations of motion because of having to use a local reference frame fixed to a flexible body. The following method is remarkable from this point of view.

The *absolute nodal coordinate* formulation introduces large displacements of finite elements relative to the global reference frame without using any local frame. The elements employ finite slopes as nodal variables and are generalizations of ordinary finite elements that use infinitesimal slopes. In contrast to other large deformation formulations, here the equations of motion contain constant mass matrices and generalized gravity forces as well as zero centrifugal and Coriolis inertia forces. The only nonlinear term is a vector of elastic forces.

This approach allows applying known abstractions of real elastic bodies: Euler–Bernoulli beams [1], Timoshenko beams and more general models [8] based on continuum mechanics as well as Kirchhoff and Mindlin plate theories.

An implementation of a more general plate finite element with the help of absolute nodal coordinate formulation was proposed by Mikkola and Shabana [7]. In their paper, three-dimensional (3D) shape functions were used and the 48-d.o.f. plate element was, in fact, a solid body. The authors applied solid mechanics theory to obtain elastic forces.

In this paper, we consider thin plates only and propose several 2D plate elements [3,4]: rectangular ones with 48 and 36 d.o.f. as well as a triangle 27-d.o.f. one. These are the direct generalizations of the 16-, 12- and 9-d.o.f. elements usually applied in the finite element method. We use Kirchhoff plate theory with nonlinear strain-displacement relationships to obtain elastic forces as well as differential geometry of parameterized surfaces to calculate mid-plane deformations, transverse curvatures and twist.

The proposed elements are able to correctly represent large overall motion because its shape functions contain a full set of rigid-body modes. It also can represent large deformations due to nonlinear strain-displacement relationships used in this paper.

2. Thin beam element using the absolute nodal coordinate formulation

In this section, we consider a way to use absolute nodal coordinate formulation as a generalization of the finite element approach. In this section we describe it for a 2D beam, a generalization of this approach for plates is given in Section 4.

Let us consider an Euler–Bernoulli beam element of length ℓ in Fig. 1. The displacements y_0 , y_ℓ and the slopes y'_0 , y'_ℓ of its two nodes are assumed infinitesimal. Its displacement field can be represented as

$$y(x) = s_1(x)y_0 + s_2(x)y'_0 + s_3(x)y_\ell + s_4(x)y'_\ell,$$

where $s_1, \dots, s_4$ are the beam shape functions [13].

In order to specify arbitrary displacements of a beam element we can parameterize the beam centerline using the arc coordinate $p \in [0, \ell]$ and introduce two displacement fields $x(p)$ and $y(p)$ instead of $y(x)$, as shown in Fig. 2.

Then, introducing the displacement and slope vectors at the end points

$$\mathbf{r}_0 = \{x_0 \ y_0\}^T = \mathbf{e}_1, \quad \mathbf{r}'_0 = \{x'_0 \ y'_0\}^T = \mathbf{e}_2,$$

$$\mathbf{r}_\ell = \{x_\ell \ y_\ell\}^T = \mathbf{e}_3, \quad \mathbf{r}'_\ell = \{x'_\ell \ y'_\ell\}^T = \mathbf{e}_4,$$

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