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ScienceDirect
Journal of Computational and Applied Mathematics 215 (2008) 378–389
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
www.elsevier.com/locate/cam

Second order blended multidimensional upwind residual distribution scheme for steady and unsteady computations☆

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Received 12 July 2005

Abstract

Multidimensional upwind residual distribution (RD) schemes have become an appealing alternative to more widespread finite volume and finite element methods (FEM) for solving compressible fluid flows. The RD approach allows to construct nonlinear second order and non-oscillatory methods at the same time. They are routinely used for steady state calculations of the complex flow problems, e.g., 3D turbulent transonic industrial-type simulations [H. Deconinck, K. Sermeus, R. Abgrall, Status of multidimensional upwind residual distribution schemes and applications in aeronautics, AAIA Paper 2000-2328, AIAA, 2000; K. Sermeus, H. Deconinck, Drag prediction validation of a multi-dimensional upwind solver, CFD-based aircraft drag prediction and reduction, VKI Lecture Series 2003-02, Von Karman Institute for Fluid Dynamics, Chausée do Waterloo 72, B-1640 Rhode Saint Genèse, Belgium, 2003].

Despite its maturity, some problems are still present for the nonlinear schemes developed up to now: namely a poor iterative convergence for the transonic problems and a decrease of accuracy in smooth parts of the flow, caused by a weak L_2 instability [M. Ricchiuto, Construction and analysis of compact residual discretizations for conservation laws on unstructured meshes. Ph.D. Thesis, Université Libre de Bruxelles, Von Karman Institute for Fluid Dynamics, 2005].

We have developed a new formulation of a blended scheme between the second order linear LDA [R. Abgrall, M. Mezine, Residual distribution scheme for steady problems, 33rd Computational Fluid Dynamics course, VKI Lecture Series 2003-05, Von Karman Institute for Fluid Dynamics, Chausée do Waterloo 72, B-1640 Rhode Saint Genèse, Belgium, 2003] scheme and the first order N scheme. The blending coefficient is based on a simple shock capturing operator and it is properly scaled such that second order accuracy is preserved. The approach is extended to unsteady flows problems using consistent formulation of the LDA scheme with the mass matrix [M. Mezine, M. Ricchiuto, R. Abgrall, H. Deconinck, Monotone and stable residual distribution schemes on prismatic space–time elements for unsteady conservation laws, 33rd Computational Fluid Dynamics Course, Von Karman Institute for Fluid Dynamics, 2003; R. Abgrall, M. Mezine, Construction of second order accurate monotone and stable residual distribution schemes for unsteady flow problems, J. Comput. Phys. 188 (2003) 16–55]. For the time integration, a three point backward scheme is selected for its accuracy and robustness and the shock capturing operator is modified appropriately, to handle moving shocks.

☆ This work was partially supported by the Research Plan MSM of the Czech Republic No. 6840770003.

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doi:10.1016/j.cam.2006.03.046

We present a numerical solution of several challenging test cases involving the solution of the Euler equations from the subsonic to the hypersonic regime. All the tests shows very good accuracy, robustness and convergence properties.

MSC: 65M99; 35L65; 76M10; 76M12

Keywords: Residual distribution scheme; Fluctuation splitting scheme; Steady problems; Unsteady problems; Finite volume method; Finite element method; Transonic flows; Shock capturing; Mass matrix; Implicit method

1. Introduction

The class of compact residual distribution (RD) schemes has become an appealing alternative for solving fluid flows to other approaches, as e.g., finite volume (FV) and finite element methods (FEM). RDS combine the second order of accuracy with non-oscillatory properties on a very compact stencil [8]. They are routinely used for steady state calculations of a complex flow problems, e.g., 3D turbulent transonic industrial-type simulations [10,20].

Among linear RD schemes, probably two of the best known schemes are the LDA and the N scheme. The LDA scheme is second order accurate scheme and it is successfully used for subsonic computations. It is not suitable for the transonic regimes, because of the oscillatory resolution of the shock waves. However, due to its upwinding character, it contains enough dissipation for medium transonic velocities. On the other hand, the N scheme is a first order, positive scheme. An example of a second order and non-oscillatory scheme is the nonlinear B scheme [10,3]. It employs a blending technique taking advantage of both the N and the LDA schemes. It performs very robustly with monotone shock capturing and it can be used for large scale applications [20]. Another approach was chosen for construction of the second order and positive N-modified (also called PSI or limited N) scheme [3]. Despite robustness of the nonlinear schemes, both the B and the N-modified schemes suffer from rather poor convergence to the steady state solution. For the implicit calculations approximation of the Jacobian by the first order scheme has to be taken, otherwise CFL numbers of order units have to be used.

The aim of the paper is to present another blending technique, which uses a shock capturing operator for blending the LDA and the N scheme. In the shock regions we use locally the N scheme for its monotone shock capturing, while in all the other regions we use the LDA scheme, because of its high accuracy. Advantage is, that the LDA scheme contains enough dissipation to resolve smooth flows and contacts discontinuities in a stable manner. The shock capturing operator is scaled such that in the smooth regions the N scheme introduces an error of higher order. Since our definition of the blending coefficient is smooth, the full Jacobian can be taken for the implicit method, which noticeably speeds up the convergence rate.

Unfortunately, a straightforward extension of the RD schemes for unsteady computations gives only first order accuracy, as was shown e.g. in [15,16,2]. The space and time derivatives cannot be written separately as they are coupled by a FEM type mass matrix. Since this matrix is not a M-matrix, simple inversion leads to an oscillatory scheme, even if the underlying spatial scheme was shown to be non-oscillatory. A first attempt was to use the flux corrected transport (FTC) method to construct a non-oscillatory scheme, but it was only moderately successful [15,13,14]. An another approach is the construction of the space–time scheme and to apply the RD schemes in space–time [6,7,16,2]. Unfortunately, nonlinear versions of these schemes also suffer from erratic convergence in dual (or pseudo) time for transonic computations. There is also a version of the N-limited space–time scheme for arbitrary time step [16], but taking a CFL number higher than order of decade practically prevents convergence in pseudo-time. Moreover this version uses twice more unknowns, which can bring some difficulties for solving large 3D problems.

For time dependent problems we present the version of the blended scheme, which uses the spatial LDA scheme with consistent mass matrix and as a non-oscillatory scheme the N scheme with the lumped mass matrix. Both methods employs the three point backward time integration scheme in the physical time. The resulting system of nonlinear equations is solved in dual time by mean of a explicit or implicit time integration. The shock capturing operator is modified by inclusion of the time derivative of the pressure to account for unsteady shocks. Since the formulation of both the LDA and N scheme is stable for arbitrary physical time step, we expect, that the resulting scheme Bx can be used for industrial-type of simulations with high CFL numbers.