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A surface capturing method for the efficient computation of steady water waves☆

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Abstract

A surface capturing method is developed for the computation of steady water–air flow with gravity. Fluxes are based on artificial compressibility and the method is solved with a multigrid technique and line Gauss–Seidel smoother. A test on a channel flow with a bottom bump shows the accuracy of the method and the efficiency of the multigrid solver.

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1. Introduction

In the design of ships and offshore structures, computations of steady water flow play an important role, e.g., computations of the wave pattern and friction drag of ships can help in optimising ship designs for low drag. To enable efficient design, these computations must be fast and accurate. A well-known technique for computing steady flows is to time-march the unsteady flow equations to steady state. But water flows with free surfaces and gravity effects need a long time to reach a steady state, as they show traveling waves that damp out very slowly. So when large-scale 3D water flows are computed, the size and the complexity of the flows that can be computed is reduced significantly. Therefore, more efficient solution techniques are highly desirable and often used [5,6].

Most existing surface-fitting methods, i.e., methods that model the free surface by deforming the mesh to fit the water surface, are equipped with efficient steady solvers. But if the free surface is modelled with a surface capturing method, like the volume-of-fluid or level set technique, time marching is still the most widely used solution method. A great advantage of surface capturing techniques is that they can handle nonlinear steep waves and near-breaking waves, as well as complex geometries near the water surface. But to fully use this advantage in the computation of steady flow, an efficient solution method is needed.

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We are developing a method for 2D water flow based on steady flow equations. Both the water flow and the air flow above it are modelled. A modified volume-of-fluid capturing technique is used to find the water–air interface and the system is solved with a multigrid method. The flux function is based on artificial compressibility.

2. Flow equations

In order to get a flow discretisation that can be solved easily with multigrid, we base our flow equations on conservation laws only. The water–air interface appears as a mixture zone, with a smooth transition from water to air. The flow of this mixture satisfies the Navier–Stokes equations, just like the pure fluids; therefore, the same equations are valid everywhere in the domain, as long as the bulk density is properly defined. To distinguish between the fluids, we add a mass conservation equation for one of the fluids. Water and air are both considered incompressible: they have constant densities. Defining α as the volume fraction of water, the mixture density is $\rho = \rho_{\mathrm{w}}\alpha + \rho_{\mathrm{a}}(1-\alpha)$. Substituting this relation in the steady compressible (variable-density) 2D Navier–Stokes equations with gravity yields the following, incompressible flow equations:

$$
\begin{aligned}
&\frac{\partial}{\partial x}(u) + \frac{\partial}{\partial y}(v) = 0 \quad \text{(tot. mass)},\\
&\frac{\partial}{\partial x}(p+\rho u^2) + \frac{\partial}{\partial y}(\rho u v) = \frac{\partial}{\partial x}(\mu u_x) + \frac{\partial}{\partial y}(\mu u_y) \quad (x\text{-mom.}),\\
&\frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(p+\rho v^2) = \frac{\partial}{\partial x}(\mu v_x) + \frac{\partial}{\partial y}(\mu v_y) - \rho g \quad (y\text{-mom.}),\\
&\frac{\partial}{\partial x}(u\alpha) + \frac{\partial}{\partial y}(v\alpha) = 0 \quad \text{(water mass)}.
\end{aligned} \tag{1}
$$

As opposed to single-fluid incompressible flow, the density is not constant, so it cannot be divided out in the momentum equations. However, it still disappears from both continuity equations: total mass conservation has its standard form $u_x + v_y = 0$ and mass conservation for the water reduces to a volume-of-fluid equation. So we have a system of equations that is equivalent to volume-of-fluid, but that is completely in conservation form.

3. Flux function

System (1) is discretised with a cell-centred finite-volume technique. The states left and right of the cell faces are taken equal to the state in the cell centre: a first-order reconstruction. Then a flux function is used to compute the flux across the cell face from these two states. The flux function is split in a convective and a diffusive part, to independently control the stability of these parts. And because of the splitting, different boundary conditions can be assigned for convection and diffusion, consistent with the fluxes.

3.1. Linearised Osher convective flux

We discretise the convective part of the flux with the artificial compressibility technique [2,7], that guarantees stability: in the time-dependent flow equations, artificial time derivatives are added to the continuity equations. The resulting hyperbolic system is used to define a Riemann flux function, which is substituted back into the steady flow equations. These are then solved directly with the multigrid technique. Such a technique is also used in [3].

The two-fluid artificial compressibility equations are found by assuming that the densities of the pure fluids in the continuity equations have time derivatives $(\rho_{\mathrm{w}})_t = p_t/c_{\mathrm{w}}^2$, $(\rho_{\mathrm{a}})_t = p_t/c_{\mathrm{a}}^2$ (but zero gradients!). The parameters c_{w} and c_{a}, with the dimension of velocity, can be chosen freely. We choose $\rho_{\mathrm{w}}c_{\mathrm{w}}^2 = \rho_{\mathrm{a}}c_{\mathrm{a}}^2 = c^2$, with a constant c, to simplify the resulting equations. Substituting this in the time-dependent version of (1) and taking only the