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Numerical solution of Volterra integral and integro-differential equations of convolution type by using operational matrices of piecewise constant orthogonal functions

E. Babolian[a,*], A. Salimi Shamloo[b]

[a] *Faculty of Mathematical Sciences and Computer Engineering, Teacher Training University, Tehran, Iran*
[b] *Department of Mathematics, Islamic Azad University of Tehran, Science and Research Branch, Tehran, Iran*

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Abstract

In this paper, we use operational matrices of piecewise constant orthogonal functions on the interval [0, 1) to solve Volterra integral and integro-differential equations of convolution type without solving any system. We first obtain Laplace transform of the problem and then we find numerical inversion of Laplace transform by operational matrices. Numerical examples show that the approximate solutions have a good degree of accuracy.

Keywords: Volterra integral equation; Volterra integro-differential equation; Weakly singular integral equations; Abel integral equations; Piecewise constant orthogonal functions; Operational matrices; Laplace transforms; Inversion of Laplace transform

1. Introduction

In recent years, many different methods have been used to approximate the solution of Volterra integral equations and Volterra integro-differential equations of convolution type [6,4,7,1,11]. In this paper, we first present the piecewise constant orthogonal functions, their properties and their operational matrices. Then we introduce Volterra integral equations and Volterra integral–differential equations of convolution type encountered in mechanics and physics, for example,weakly singular Abel integral equations.

2. Piecewise constant orthogonal functions

Let I be the normal interval $[0, 1)$, the system of $\{\theta_i\}$ is called piecewise constant, if θ_i for $i = 1, 2, \ldots, m$ be constant on each subinterval $[\frac{i-1}{m}, \frac{i}{m})$.

* Corresponding author.
E-mail addresses: babolian@saba.tmu.ac.ir (E. Babolian), ali-salimi@iaushab.ac.ir (A.S. Shamloo).

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Definition 2.1. An m-set of Block-Pulse functions on [0, 1) is defined as

$$\beta_i(t) = \begin{cases} 1, & \dfrac{i-1}{m} \leqslant t < \dfrac{i}{m}, \\ 0, & \text{otherwise}. \end{cases} \tag{2.1}$$

Haar functions and Walsh functions are also piecewise constant orthogonal functions. Since Haar functions and Walsh functions are linear combination of Block-Pulse functions [8], we only use Block-Pulse functions for solving problems under consideration.

Every function $f(t)$ which is square integrable in the interval [0, 1) can be expanded in terms of Block-Pulse functions series as

$$\sum_{1}^{m} f_i \beta_i(t), \tag{2.2}$$

where $f_i = \int_0^1 f(t)\beta_i(t)\,\mathrm{d}t$. Eq. (2.2) can be written as $\overrightarrow{f}^{\mathrm{T}} = \overrightarrow{C}^{\mathrm{T}}.B_m$, where $\overrightarrow{f}^{\mathrm{T}}$ is the discrete form of the continuous function, $f(t)$, and $\overrightarrow{C}^{\mathrm{T}}$ is called the wavelet coefficient. They are both column vectors, and B_m is the Block-Pulse matrix and is defined by

$$B_m = [\overrightarrow{b}_1^{\mathrm{T}}, \overrightarrow{b}_2^{\mathrm{T}}, \ldots, \overrightarrow{b}_m^{\mathrm{T}}], \tag{2.3}$$

where $\overrightarrow{b}_1^{\mathrm{T}}, \overrightarrow{b}_2^{\mathrm{T}}, \ldots, \overrightarrow{b}_m^{\mathrm{T}}$ are discrete form of the Block-Pulse bases; the discrete values are taken from the continuous curves $\beta_1(t), \beta_2(t), \ldots, \beta_m(t)$, respectively (a one-to-one correspondence between piecewise constant orthogonal functions and vectors in R^m can be defined [2]).

Note: In the following sections we will not use the vector symbols for simplicity.

3. Error in piecewise constant orthogonal functions approximation

From [8], if we expand a function $f(t)$ by Block-Pulse functions, then the ith Fourier coefficient is given by

$$f_{\beta_i} = \frac{(f(t), \beta_i)}{\|\beta_i\|} = \sqrt{m} \int_{(i-1)/m}^{i/m} f(t)\,\mathrm{d}t, \tag{3.1}$$

where $(f(t), \beta_i) = \int_0^1 f(t)\beta_i(t)\,\mathrm{d}t$ and $\|\beta_i\| = (\int_0^1 (\beta_i)^2\,\mathrm{d}t)^{1/2}$.

By virtue of the mean value theorem, we have

$$f_{\beta_i}^2 = \frac{1}{m} f^2(t_i), \quad t_i \in \left[\frac{i-1}{m}, \frac{i}{m}\right]. \tag{3.2}$$

The error when a differentiable function $f(t)$ is represented in a series of piecewise constant orthogonal functions over subinterval $[\frac{i-1}{m}, \frac{i}{m})$ is $e_i(t) = f_{\beta_i}\beta_i(t) - f(t)$. It can be shown that

$$\|e_i\|^2 = \frac{1}{12m^2}[f'(t_i)]^2, \quad t_i \in [(i-1)/m, i/m).$$

This leads to

$$\|e(t)\| \leqslant \frac{1}{2\sqrt{3}m}\|f'\|_\infty, \tag{3.3}$$

where $e(t) = \sum_1^m f_{\beta_i}\beta_i(t) - f(t)$.