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Stability criteria for impulsive systems on time scales☆

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Abstract

In this paper we study stability of impulsive system on time scales. By using comparison method, Lyapunov function and analysis method, the asymptotic stability criteria for system with impulses at fixed times and impulses at variable times on time scales are obtained, respectively. An example is presented to illustrate the efficiency of proposed result.

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Keywords: Stability; Asymptotic stability; Time scale; Impulsive system; Lyapunov function

1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D. thesis in 1988 in order to unify continuous and discrete analysis. The book [3] is an introduction to the study of dynamic equations on time scales and the general idea of it is to prove a result for a dynamic equation where the domain is a so-called time scale, which is an arbitrary nonempty closed subset of the reals. Since impulsive systems have a wide variety of applications such as aircraft control, drug administration and threshold theory in biology, the theory of impulsive systems has developed very well and various interesting results have been reported [2,9,11,12]. Recently, though a lot of work had been done on the stability problem of dynamic equations on time scales [1,4–8], there are rare work on the stability of impulsive systems on time scales [10].

In this paper we study the stability of the impulsive systems on time scales. We use comparison result [10], Lyapunov function and analysis method to study stability of system with impulses at fixed times and impulses at variable times on time scales, respectively. In this paper, we assume that the times of impulses belong to time scale T, otherwise it is unreasonable. At last an example is presented to illustrate the efficiency of proposed result.

2. Preliminaries

Let T be a time scale (an arbitrary nonempty closed subset of the real numbers) with $t_0 \geqslant 0$ as minimal element and no maximal element.

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Definition 2.1 (*Bohner and Peterson [3]*). The mappings $\sigma, \rho : T \to T$ defined as $\sigma(t) = \inf\{s \in T : s > t\}$ and $\rho(t) = \sup\{s \in T : s < t\}$ are called jump operators.

Definition 2.2 (*Bohner and Peterson [3]*). If $\sigma(t) > t$, we say that t is right scattered (rs), while if $\rho(t) < t$ we say that t is left scattered (ls). Also, if $t < \sup T$ and $\sigma(t) = t$, then t is called right dense (rd), and if $t > \inf T$ and $\rho(t) < t$, then t is called left dense (ld).

Definition 2.3 (*Bohner and Peterson [3]*). The graininess function $\mu : T \to [0, \infty)$ is defined by

$$\mu(t) = \sigma(t) - t.$$

Definition 2.4. We define the interval $[a, b]^*$ in T by

$$[a, b]^* = \{t \in T : a \leqslant t \leqslant b\}.$$

Open intervals and half-open intervals are defined accordingly.

Definition 2.5 (*Bohner and Peterson [3]*). Assume $f : T \to R$ is a function and let $t \in T$. Then we define $f^{\Delta}(t)$ to be the number with the property that given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t - \delta, t + \delta) \cap T$ for some $\delta > 0$) such that

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \leqslant \varepsilon|\sigma(t) - s|$$

for all $s \in U$ we call $f^{\Delta}(t)$ the delta derivative of f at t.

Definition 2.6 (*Bohner and Peterson [3]*). A function $f : T \to R$ is called rd-continuous provided it is continuous at right-dense points in T and its left-sided limits exist (finite) at left-dense points in T. The set of rd-continuous functions $f : T \to R$ will be denoted by C_{rd}.

Definition 2.7 (*Lakshmikantham and Vatsala [10]*). Define for $V \in C_{\mathrm{rd}}[T \times R^n, R^+]$ then we define $V^{\Delta}(t, x(t))$ to mean that, given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t - \delta, t + \delta) \cap T$ for some $\delta > 0$) such that

$$|[V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t)))] - \mu(t, s)f(t, x(t)) - \mu(t, s)V^{\Delta}(t, x(t))| \leqslant \varepsilon|\mu(t, s)|$$

for all $s \in U$ of f at t.

We also define $D^+V^{\Delta}(t, x(t))$ to mean that, given any $\varepsilon > 0$, there is a right neighborhood $U_\varepsilon \subset U$ of t such that

$$\frac{1}{\mu(t, s)}[V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t, s)f(t, x(t))] < D^+V^{\Delta}(t, x(t)) + \varepsilon$$

for each $s \in U_\varepsilon$, $s > t$.

Definition 2.8. V is said to belong to class V_0 if

(i) V is continuous in $(t_{k-1}, t_k]^* \times R^n$ and for each $x \in R^n$, $k = 1, 2, \ldots, \lim_{(t,y)\to(t_k^+,x)} V(t, y) = V(t_k^+, x)$ exists.
(ii) V is locally Lipschitzian in x and $V(t, 0) = 0$.

V is said to belong to class V_1 if $V \in V_0$ is continuously delta differentiable in $(t_{k-1}, t_k]^* \times R^n$.

Definition 2.9 (*Yang [11]*). If the functions $\tau_k(x) : S(\rho) \to R^+, k \in N$ are continuous and $0 = \tau_0(x) < \tau_1(x) < \tau_2(x) < \cdots$, $\lim_{k\to\infty} \tau_k(x) = \infty$ then we define the sets

$$G_k = \{(t, x) \in T \times R^n : \tau_{k-1}(x) < t < \tau_k(x)\} \quad \text{and} \quad G = \bigcup_{k=1}^{\infty} G_k.$$