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On exact travelling wave solutions for two types of nonlinear $K(n,n)$ equations and a generalized KP equation

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Abstract

In this paper, we study two types of genuinely nonlinear $K(n,n)$ equations and a generalized *KP* equation. By developing a mathematical method based on the reduction of order of nonlinear differential equations, we derive general formulas for the travelling wave solutions of the three equations. The compactons, solitary patterns, solitons and periodic solutions obtained are expressed analytically. It is shown that the y and z components of the wave number vectors in the travelling wave solutions of the generalized *KP* equation remain free and arbitrary constants. The work generalizes the known results of travelling wave solutions for the three equations.
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1. Introduction

Recently, intensive research has been conducted to study the following $K(n,n)$ evolution equation

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0, \quad n \geqslant 1, \tag{1}$$

which describes the role of nonlinear dispersion in the formation of patterns in liquid drops (see [10,13]). The studies, as presented in [5–10,12–14], discovered that nonlinear dispersion can compactify solitary waves and generate compactons: solitons with finite wavelength or robust soliton-like solutions characterized by the absence of infinite wings. The discovery of compactons that collide elastically and vanish identically outside a finite core region was made in [10] to specify and establish a scientific explanation that nonlinear dispersion leads to qualitative changes [5] in the nature of some genuinely nonlinear phenomena. The tri-Hamiltonian duality between solitons and compactons was reported in Olver and Rosenau [5]. Rosenau and coworkers [5,6,8,10] found that the collision of two compactons results in the creation of low-amplitude compacton and anticompacton pairs, and they reemerge with same coherent shape.

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Various attempts to study Eq. (1) have been made in recent years. Rosenau and Hyman [10] used the pseudo-spectral method in space and a variable order, variable time-step Adams–Basford–Moulton method in time to study Eq. (1). Wazwaz [12] made use of the Adomian decomposition method to study Eq. (1), particularly, the cases $K(2,2)$ and $K(3,3)$, and derived a general formula of compacton solutions of Eq. (1) for $n>1$. Ismail and Taha [3] developed a finite difference method and a finite element method to study Eq. (1) for $n=2$ and 3, and obtained numerical solutions for one compacton which were then compared with the exact solutions to assess the accuracy of the methods. It was shown in [3] that the numerical solutions agreed very well with the exact solutions and that the compactons exhibited true soliton behavior. For more details of the methods to acquire compactons and soliton solutions for nonlinear evolution equations, the reader is referred to [1–3,9,11–15] in which many analytical and numerical methods such as the pseudo-spectral method, the Galerkin method, the finite difference method, the sine–cosine ansatz, and the tanh method are presented.

Rosenau [9] and Wazwaz [13] investigated the following models:

$$u_t + a(u^{n+1})_x + [u(u^n)_{xx}]_x = 0, \quad a>0,\ n\geqslant 1, \tag{2}$$

and

$$\{u_t + a(u^{n+1})_x + [u(u^n)_{xx}]_x\}_x + \nabla_{\top}^2 u = 0, \quad a>0,\ n\geqslant 1, \tag{3}$$

where $\nabla_{\top}^2 = (k(k-1)/k!)\partial_y^2 + (k(k-1)(k-2)/k!)\partial_z^2,\ k=2$ or 3, in which k represents the dimension of the spatial domain. Rosenau [9] regarded Eq. (2) as another variant of the $K(n,n)$ model which was shown to describe the dispersion of dilute suspensions [9] for $n=1$. Eq. (3) is a generalized form of the well-known KP equation. In [9], some meaningful results were obtained to explore a number of formal mathematical extensions of solitons supporting equations with the aim of producing compact dispersive structures in higher dimensions, and several non-travelling wave solutions for the generalized KP equation were constructed by a mathematical transformation formula.

Motivated by the form of Eq. (3), we write another generalized type of KP equation expressed in the form

$$\{u_t + a(u^{n+1})_x + [u(u^n)_{xx}]_x\}_x + b_1 u_{yy} + b_2 u_{zz} = 0, \quad a\neq 0,\ n\neq 0, \tag{4}$$

where constants b_1 and b_2 satisfy $b_1^2 + b_2^2 \neq 0$. Obviously, Eq. (4) reduces to Eq. (3) for $b_1 = k(k-1)/k!$ and $b_2 = b_1(k-2)$.

In this paper, we further develop the work in [10,9,12,13] for the study of Eqs. (1), (2) and (4). By using a mathematical technique different from those in previous work [5–10,12–15], we obtain general formulas for travelling wave solutions with wave variable $\xi = \mu(x-ct)$ or $\xi = \mu x + \eta y + \zeta z - ct$ for three nonlinear Eqs. (1), (2) and (4). For $K(n,n)$ equations (1) and (2), we find that the exponent n and a, positive or negative, determine directly the physical structures of solutions such as compactons, solitons, solitary patterns and periodic solutions.

2. Solving $K(n,n)$ equation with positive and negative n

Firstly, we consider the solution of the equation

$$\left(\frac{\mathrm{d}W}{\mathrm{d}z}\right)^2 = a_0 - b_0 W^2, \tag{5}$$

where $a_0 \neq 0$ and $b_0 \neq 0$ are constants. When $b_0 > 0$, Eq. (5) admits two solutions

$$W_1 = \pm\sqrt{\frac{a_0}{b_0}}\sin[\sqrt{b_0}(z+A)], \quad W_2 = \pm\sqrt{\frac{a_0}{b_0}}\cos[\sqrt{b_0}(z+A)], \tag{6}$$

where A is an arbitrary constant.

When $b_0 < 0$, noticing that $\cosh^2 z - \sinh^2 z = 1$, we know that Eq. (5) has two solutions of the form

$$W_3 = \pm\sqrt{-\frac{a_0}{b_0}}\sinh[\sqrt{-b_0}(z+A)], \quad W_4 = \pm \mathrm{i}\sqrt{-\frac{a_0}{b_0}}\cosh[\sqrt{-b_0}(z+A)], \tag{7}$$

where $\mathrm{i} = \sqrt{-1}$.

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