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Stochastic optimal control and algorithm of the trajectory of horizontal wells

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Abstract

This paper presents a nonlinear, multi-phase and stochastic dynamical system according to engineering background. We show that the stochastic dynamical system exists a unique solution for every initial state. A stochastic optimal control model is constructed and the sufficient and necessary conditions for optimality are proved via dynamic programming principle. This model can be converted into a parametric nonlinear stochastic programming by integrating the state equation. It is discussed here that the local optimal solution depends in a continuous way on the parameters. A revised Hooke–Jeeves algorithm based on this property has been developed. Computer simulation is used for this paper, and the numerical results illustrate the validity and efficiency of the algorithm.

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1. Introduction

The research on designing the trajectory of horizontal wells considerably developed over these last years. Many methods in dealing with specific problems have been put forward, respectively. In general, the well path is a three-dimensional curve that reaches a given target from a given starting location subject to several constraints. There are some well planning programs available commercially to solve the problem of designing an appropriate trajectory of a horizontal well. However, those methods belong to the category of trial-and-error or human–computer interaction essentially, in which the identification of some control parameters depends to a great extent on the designers' experience and intuition. In recent years, very few references have discussed the horizontal well planning in the mathematical literature. Foreign and domestic experts mainly put forward nonlinear programming models [2,4,6], a fuzzy model [9] and an optimization model [8]. In fact, there are more unknown parameters for complete well than there are defining equations. Consequently, the problem of finding a well path is underdetermined in existing results. Due to the effects of some factors such as stratum and tools, the real trajectory of horizontal wells is deviant from the theoretically optimal one in drilling. But such perturbations have been ignored or only been given a little qualitative consideration in the previous designs. If the parameters provided by the optimal design are applied into practice, the trajectory may not

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achieve optimal, or even deviates from the target. Therefore, we establish a nonlinear, multi-phase and stochastic control system of the trajectory of horizontal wells based on the dynamic model [8] Stochastic control is the study of dynamical systems subject to random perturbations and which can be controlled in order to optimize some performance criterion. Our chief concern is to derive some tractable characterization of the value function and optimal control. This article is intended to prove the sufficient and necessary conditions of optimal solution and that the optimal solution depends in a continuous way on the parameters (perturbations). In addition, a revised Hooke–Jeeves algorithm is proposed and the corresponding software is programmed to calculate the practical problems. The numerical results demonstrate the correctness and effectiveness of the stochastic control model and algorithm.

The rest of this paper is organized as follows. Section 2 consist of the problem description and the mathematical model. The existence and uniqueness of the solution of stochastic differential equation are discussed in Section 3. Section 4 give the key results on the characterization of optimality. In Section 5, the parametric nonlinear stochastic programming problem is introduced and some important properties are proved. Finally, the optimization algorithm is proposed to solve the nonlinear stochastic programming with a numerical issue and the conclusions of the paper are mentioned.

2. Problem formulation

As is shown in Fig. 1, the trajectory of a horizontal well can be described in a cartesian coordinate system having its origin at the initial point (Kick-off Point), with x-axis representing North, y-axis representing East, and z-axis representing the vertical depth. Any point on the curve is completely described by its inclination φ , azimuth φ , and coordinates x, y, z. In order to simplify the problem, we idealize the trajectory of horizontal wells to be a combination of alternately n constant-curvature smooth quasi-helix segments. Tool-face angle w and curvature w are key parameters to drill a horizontal well, which are governed by the general build-up rate of bottle-hole assembly (BHA) in the drilling operation. Under such assumptions, the rate of change of inclination w0 and the rate of change of azimuth w1 obey the following rules, respectively,

$$K_{\varphi} = K \cos w, \quad K_{\phi} = \frac{K \sin w}{\sin \varphi}.$$

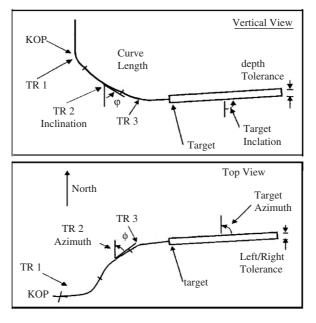


Fig. 1. Horizontal Well's terminology.

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