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Philos-type oscillation criteria for Emden–Fowler neutral delay differential equations

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Abstract

Philos-type oscillation criteria are established for the Emden–Fowler neutral delay differential equation

$$[|x'(t)|^{\gamma-1}x'(t)]' + q_1(t)|y(t-\sigma)|^{\alpha-1}y(t-\sigma) + q_2(t)|y(t-\sigma)|^{\beta-1}y(t-\sigma) = 0, \quad t \geqslant t_0,$$

where $x(t) = y(t) + p(t)y(t-\tau)$. The results obtained here essentially improve some known results in the literature. In particular, two interesting examples that point out the importance of our results are also included.
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1. Introduction

In this paper, we study the problem of oscillation of the Emden–Fowler neutral delay differential equation

$$[|x'(t)|^{\gamma-1}x'(t)]' + q_1(t)|y(t-\sigma)|^{\alpha-1}y(t-\sigma) + q_2(t)|y(t-\sigma)|^{\beta-1}y(t-\sigma) = 0, \quad t \geqslant t_0, \tag{1.1}$$

where $x(t) = y(t) + p(t)y(t-\tau)$. In what follows we assume that

(A1) τ and σ are nonnegative constants, α, β and γ are positive constants with $0 < \alpha < \gamma < \beta$;

(A2) $q_1, q_2 \in C([t_0, \infty), \mathbb{R}^+)$, $\mathbb{R}^+ = (0, \infty)$;

(A3) $p \in C([t_0, \infty), \mathbb{R})$, and $-1 < p_0 \leqslant p(t) \leqslant 1$, p_0 constant.

Our attention is restricted to those solutions $y = y(t)$ of (1.1) which exist on some half-line $[t_y, \infty)$ with $\sup\{|y(t)| : t \geqslant T\} > 0$ for any $T \geqslant t_y$, and satisfy (1.1). We make the standing hypothesis that (1.1) does possess such a solution [11]. As usual, a solution of (1.1) is said to be oscillatory if the set of its zeros is unbounded from above, otherwise it is called nonoscillatory. Eq. (1.1) is called oscillatory if all of its solutions are oscillatory. We say that Eq. (1.1) satisfies the superlinear condition if $q_1(t) \equiv 0$ and it satisfies the sublinear condition if $q_2(t) \equiv 0$.

We note that second order neutral delay differential equations are used in many fields such as vibrating masses attached to an elastic bar and some variational problems, see [11].

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In the last decades, there has been an increasing interest in obtaining sufficient conditions for the oscillation and/or nonoscillation of second order linear and nonlinear neutral delay differential equations (see, for example, [1–3,5,7–10,14,15,17–21,23] and the references therein). Let us consider the second order neutral delay differential equation

$$[y(t) + p(t)y(t-\tau)]'' + q(t)f(y(t-\sigma)) = 0. \tag{1.2}$$

To the best of our knowledge, almost all of the known results obtained for (1.2) required the assumption that the function $f(y)$ satisfies $f'(y) \geqslant k > 0$ or $f(y)/y \geqslant k > 0$ for $y \neq 0$, (see, [3,5,7–10,14,15,17,18,23]), which is not applicable for $f(y) = |y|^{\nu-1}y$, the classical Emden–Fowler case. Recently, the results of Atkinson [4] and Belohorec [6] for second order ordinary differential equation were extended to (1.2) by Wong [21] under the assumption that the nonlinear function f satisfies the sublinear condition

$$0 < \int_{0+}^{\varepsilon} \frac{\mathrm{d}u}{f(u)}, \quad \int_{0-}^{-\varepsilon} \frac{\mathrm{d}u}{f(u)} < \infty \quad \text{for all } \varepsilon > 0,$$

as well as the superlinear condition

$$0 < \int_{\varepsilon}^{\infty} \frac{\mathrm{d}u}{f(u)}, \quad \int_{-\varepsilon}^{-\infty} \frac{\mathrm{d}u}{f(u)} < \infty \quad \text{for all } \varepsilon > 0.$$

Also it will be of great interest to find some oscillation criteria for the special case for (1.2), even for the Emden–Fowler equation

$$[y(t) + p(t)y(t-\tau)]'' + q(t)|y(t-\sigma)|^{\nu-1}y(t-\sigma) = 0, \quad \nu > 0. \tag{1.3}$$

This problem was posed by Wong [21, Remark d]. As an affirmative answer to it, Saker [19], Saker and Manojlovič [20] have given some oscillation criteria for (1.2) and (1.3). However, these results cannot be applied to (1.1). Therefore, in the present paper, by using the averaging technique [13,16,22] and the generalized Riccati transformation [24], we shall establish Philos-type oscillation criteria [16] for (1.1). Our theorems essentially improve some known results in [19,20]. In particular, two interesting examples that point out the importance of our results are also included.

2. Main results

In this section, we shall establish Philos-type oscillation criteria for (1.1) under the cases when $0 \leqslant p(t) \leqslant 1$ and $-1 < p_0 \leqslant p(t) \leqslant 0$, which extend the results in [13,16,22] to (1.1). It will be convenient to make the following notations in the remainder of this paper. Define

$$\mu = \min\left\{\frac{\beta-\alpha}{\beta-\gamma}, \frac{\beta-\alpha}{\gamma-\alpha}\right\}, \quad k = \frac{1}{(\gamma+1)^{\gamma+1}},$$

$$Q_1(t) = \mu[1-p(t-\sigma)]^{\gamma}[q_1^{\beta-\gamma}(t)q_2^{\gamma-\alpha}(t)]^{1/(\beta-\alpha)},$$

$$Q_2(t) = \mu[q_1^{\beta-\gamma}(t)q_2^{\gamma-\alpha}(t)]^{1/(\beta-\alpha)}.$$

In order to present our theorems, we first introduce, following Philos [16], the function class $\Im$ which will be extensively used in the sequel. Namely, let $D_0 = \{(t,s) \in \mathbb{R}^2 : t > s \geqslant t_0\}$ and $D = \{(t,s) \in \mathbb{R}^2 : t \geqslant s \geqslant t_0\}$. We say that the function $H \in C(D, \mathbb{R})$ belongs to the class $\Im$, denoted by $H \in \Im$, if

(H1) $H(t,t) = 0$ for $t \geqslant t_0$, $H(t,s) > 0$ on $(t,s) \in D_0$;

(H2) H has a continuous and nonpositive partial derivative on D_0 with respect to the second variable, such that

$$\frac{\partial}{\partial s} H(t,s) = -h(t,s)H(t,s) \quad \text{for } (t,s) \in D_0,$$

where $h \in C(D, \mathbb{R})$.

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