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# A posteriori error bound methods for the inclusion of polynomial zeros[☆]

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**Abstract**

Using Carstensen's results from 1991 we state a theorem concerning the localization of polynomial zeros and derive two a posteriori error bound methods with the convergence order 3 and 4. These methods possess useful property of inclusion methods to produce disks containing all simple zeros of a polynomial. We establish computationally verifiable initial conditions that guarantee the convergence of these methods. Some computational aspects and the possibility of implementation on parallel computers are considered, including two numerical examples. A comparison of a posteriori error bound methods with the corresponding circular interval methods, regarding the computational costs and sizes of produced inclusion disks, were given.
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## 1. Localization of zeros and a posteriori error bound methods

Before running any locally convergent iterative method for the simultaneous determination of polynomial zeros, it is necessary to apply a multi-stage globally convergent composite algorithm that can provide sufficiently close initial approximations (see, e.g., [3,20,27]). The localization of zeros is an important part of this composite algorithm; a numerous references have been devoted to this subject, including famous books [16,13, Chapter 6]. One of the most beautiful results in this topic, connected with Gerschgorin's theorem and localization of zeros, is due to Carstensen [5] (Section 1). Adapting this result we can state computationally verifiable initial conditions for a number of simultaneous methods for finding polynomial zeros (see [24,25]) and construct iterative methods that produce disks in the complex plane containing the sought zeros (Section 2). The centers of these disks are calculated by some suitable iterative method with fast convergence and they present approximations to the zeros. The radii are evaluated using a corollary of Carstensen's result and give a posteriori error bounds related to these approximations.

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In this paper we combine good properties of iterative methods with fast convergence and a posteriori error bounds to construct efficient inclusion methods for polynomial complex zeros. Simultaneous determination of both centers and radii leads to iterative error bound methods which have very convenient inclusion property at each iteration. This class of methods possesses a high computational efficiency since it requires less numerical operations compared to usual interval methods realized in interval arithmetic. Numerical experiments demonstrate equal or even better convergence behavior of these methods than the corresponding interval methods realized in circular complex arithmetic (Section 3). In this paper the main attention is devoted to the construction of inclusion error bound methods together with its efficient implementation and initial conditions for the guaranteed convergence, and to the study of the convergence rate of a posteriori error bounds.

Let us return to Carstensen's result. Let $P(z)=z^n+a_1z^{n-1}+\cdots+a_{n-1}z+a_n$ $(a_i\in\mathbb{C})$ be a monic polynomial and let

$$W(z_i)=\frac{P(z_i)}{\prod_{\substack{j=1\\ j\neq i}}^{n}(z_i-z_j)}\quad (i\in I_n:=\{1,\dots,n\})$$

be Weierstrass' correction, where $z_1,\dots,z_n$ are distinct approximations to the simple zeros $\zeta_1,\dots,\zeta_n$ of $P$. Sometimes, we will write $W(z_i)=W_i$. Starting from Carstensen's result [6] which are concerned with the best Gerschgorin disks in a class of problems dealing with Weierstrass' corrections, we obtain the sharpest inclusion disks in the mentioned class given in the following theorem (see [25, Section 1.2, 29] for more details):

**Theorem 1** (*Carstensen [5]*). *Let $\eta_i:=z_i-W_i\in\mathbb{C}\backslash\{z_1,\dots,z_n\}$ and set*

$$\delta_i:=|W_i|\cdot\max_{j=1,\dots,n,\,j\neq i}|z_j-\eta_i|^{-1},\quad \sigma_i:=\sum_{j=1,\,j\neq i}^{n}\frac{|W_j|}{|z_j-\eta_i|}\quad (i\in I_n).$$

*If*

$$\sqrt{1+\delta_i}>\sqrt{\delta_i}+\sqrt{\sigma_i}\quad\text{and}\quad \delta_i+2\sigma_i<1, \tag{1}$$

*then there is exactly one zero of $P$ in the disk with center $\eta_i$ and radius*

$$r_i^*=|W_i|\frac{\delta_i+\sigma_i}{1-\sigma_i}. \tag{2}$$

**Remark 1.** The quantity $W_i$ is often called *Weierstrass' correction* since it appears in the very familiar Weierstrass' iterative method for the simultaneous determination of all simple zeros of a polynomial

$$\hat{z}_i=z_i-W_i\quad (i\in I_n), \tag{3}$$

also called the Durand–Kerner method [10,15]. Let us note that $\eta_i$ in Theorem 1 coincides with $\hat{z}_i$.

Studying the problem of calculation of zeros, it is of interest to consider simultaneously the problem of localization of zeros together with other important topics: distribution of initial approximations $z_1^{(0)},\dots,z_n^{(0)}$, their closeness and the convergence of a posteriori error bounds (shorter PEB) given by the size of inclusion regions containing zeros. An extensive research performed during the last two decades (see, e.g., [22,24,25,31]) showed that the mentioned study can be realized by using Theorem 1 and an initial condition of the form

$$w^{(0)}\leqslant c_n d^{(0)}, \tag{4}$$

where

$$w^{(m)}=\max_{1\leqslant i\leqslant n}|W(z_i^{(m)})|,\quad d^{(m)}=\min_{\substack{1\leqslant i,j\leqslant n\\ j\neq i}}|z_i^{(m)}-z_j^{(m)}|\ \ (m=0,1,\dots),$$

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