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Generalized abstract economy and systems of generalized vector quasi-equilibrium problems

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Abstract

The present paper is in two-fold. The first fold is devoted to the existence theory of equilibria for generalized abstract economy with a lower semicontinuous constraint correspondence and a fuzzy constraint correspondence defined on a noncompact/nonparacompact strategy set. In the second fold, we consider systems of generalized vector quasi-equilibrium problems for multivalued maps (for short, SGVQEPs) which contain systems of vector quasi-equilibrium problems, systems of generalized mixed vector quasi-variational inequalities and Debreu-type equilibrium problems for vector valued functions as special cases. By using the results of first fold, we establish some existence results for solutions of SGVQEPs.

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1. Introduction

The notion of an abstract economy (social system) was introduced by Debreu [10]. He proved the existence of an equilibrium point for abstract economy. For the finite number of agents, Shafer and Sonnenschein [19] and Borglin and Keiding [8] extended Debreu's result to abstract economy without order preferences. During the last two decades, many authors studied the existence of equilibrium of an abstract economy with infinite number of agents but under the compactness/paracompactness of strategy set; See for example [24,26] and references therein. In 1990, Tian [23] proved an equilibrium existence theorem for noncompact abstract economy with a countable number of agents. In the recent past, many authors studied the existence of equilibria for abstract economy with infinite number of agents; See for example [11–13,28] and references therein for paracompact/compact strategy set; for noncompact/nonparacompact strategy set we refer to [6,12,13,17] and references therein. Recently, Kim and Tan [15] and Lin et al. [18] considered abstract economy with a fuzzy constraint correspondence, known as generalized abstract economy. They established

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some existence results for an equilibrium point of generalized abstract economy under the assumption of open lower section of correspondences involved. This notion of generalized abstract economy generalizes the concept of abstract economies considered in the references given in this paper and references therein.

System of vector (quasi-) equilibrium problems (for short, SV(Q-)EP) is a unified model of several problems, for instance, system of vector (quasi-) variational inequalities (for short, SV(Q-)VI), system of vector (quasi-) optimization problems and Debreu-type equilibrium problem, also known as noncooperative game, for vector valued functions (for short, Debreu VEP). Recently, Ansari et al. [1] used SVQEP as a tool to study the existence of a solution of (Debreu VEP)(I) (See Section 3.1). They also used SVQVI to prove the existence of a solution of Debreu VEP for nonconvex but differentiable (in some sense) vector valued functions. In [2], system of generalized vector quasi-variational inequalities is used to establish the existence of solution of (Debreu VEP)(I) (See Section 3.1) for nondifferentiable and nonconvex vector valued functions.

The present paper is divided into two folds. The first fold deals with the study of existence of equilibria for generalized abstract economy with a lower semicontinuous constraint correspondence and a fuzzy constraint correspondence defined on noncompact/nonparacompact strategy set. In the second fold, we consider systems of generalized vector quasi-equilibrium problems for multivalued trifunction (for short, SGVQEPs) which contain systems of vector quasi-equilibrium problems for trifunctions, systems of mixed vector quasi-variational inequalities and Debreu-type equilibrium problems for vector valued functions as special cases. As applications of results of first fold, we establish some existence results for solutions of SGVQEPs.

2. Generalized abstract economy

For a subset Ω of a vector space, we denote by coΩ the convex hull of Ω. If Ω and Δ are subsets of a topological space $\mathscr{X}$ such that $\Omega \subseteq \Delta$, then the closure (respectively, interior) of Ω in Δ is denoted by $\mathrm{cl}_\Delta \Omega$ (respectively, $\mathrm{int}_\Delta \Omega$); In case $\Delta = \mathscr{X}$, we write clΩ and int Ω instead of $\mathrm{cl}_{\mathscr{X}}\Omega$ and $\mathrm{int}_{\mathscr{X}}\Omega$, respectively. Let $\mathscr{X}$ and $\mathscr{Y}$ be topological vector spaces and $\Phi, \Psi : \mathscr{X} \to 2^{\mathscr{Y}}$ be correspondences. Then coΦ, $\Phi \cap \Psi : \mathscr{X} \to 2^{\mathscr{Y}}$ are defined as $(\mathrm{co}\Phi)(x) = \mathrm{co}\Phi(x)$ and $(\Phi\cap\Psi)(x)=\Phi(x)\cap\Psi(x)$ for all $x \in \mathscr{X}$, respectively. For a nonempty subset V of $\mathscr{Y}$, $\Phi^{-1}(V)=\{x \in \mathscr{Y} : \Phi(x)\cap V \neq \emptyset\}$ and also $x \in \Phi^{-1}(y)$ if and only if $y \in \Phi(x)$. Φ is said to have an *open lower section* if for each $y \in \mathscr{Y}$, $\Phi^{-1}(y)$ is open in $\mathscr{X}$. We also define $\overline{\Phi}$, cl$\Phi : \mathscr{X} \to 2^{\mathscr{Y}}$ by

$$\overline{\Phi}(x) = \{y \in \mathscr{Y} : (x, y) \in \mathrm{cl}_{\mathscr{X}\times\mathscr{Y}}\, \mathrm{Gr}(\Phi)\},$$

where $\mathrm{Gr}(\Phi) = \{(x, y) \in \mathscr{X} \times \mathscr{Y} : y \in \Phi(x)\}$ denotes the graph of Φ, and

$$\mathrm{cl}\Phi(x) = \mathrm{cl}_{\mathscr{Y}}(\Phi(x)) \quad \text{for all } x \in \mathscr{X}, \text{ respectively.}$$

It is easy to see that $\mathrm{cl}\Phi(x) \subseteq \overline{\Phi}(x)$ for all $x \in \mathscr{X}$.

In a real market, any preference of a real agent could be unstable because of the fuzziness of consumers' behavior or market situations. Thus, Kim and Tan [15] introduced the fuzzy constraint correspondences in defining the following generalized abstract economy.

Let I be any set of agents (countable or uncountable). For each $i \in I$, let X_i be a nonempty set of actions available to the agent i in a topological vector space E_i and $X = \prod_{i\in I} X_i$. A *generalized abstract economy* (or *generalized game*) $\Gamma = (X_i, A_i, F_i, P_i)_{i\in I}$ [15] is defined as a family of ordered quadruples (X_i, A_i, F_i, P_i) where $A_i : X \to 2^{X_i}$ is a constraint correspondence such that $A_i(x)$ is the state attainable for the agent i at x, $F_i : X \to 2^{X_i}$ is a fuzzy constraint correspondence such that $F_i(x)$ is the unstable state for the agent i, and $P_i : X \times X \to 2^{X_i}$ is a preference correspondence such that $P_i(x, y)$ is the state preference by the agent i at (x, y). An *equilibrium* for Γ is a point $(\hat{x}, \hat{y}) \in X \times X$ such that for each $i \in I$, $\hat{x}_i \in \overline{A_i}(\hat{x})$, $\hat{y}_i \in \overline{F_i}(\hat{x})$ and $P_i(\hat{x}, \hat{y}) \cap A_i(\hat{x}) = \emptyset$.

This problem is further considered and studied in [18] with or without involving Φ-condensing correspondences.

If for each $i \in I$ and for all $x \in X$, $F_i(x) = X_i$ and the preference correspondence P_i is independent of y, that is, $P_i(x, y) = P_i(x)$ for all $x, y \in X$, our definitions of a generalized abstract economy and an equilibrium coincide with the usual definitions of an abstract economy and an equilibrium due to Shafer and Sonnenschein [19]; See also [11,21,27] and references therein.