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Fast data extrapolating

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Abstract

Data-extrapolating (extension) technique has important applications in image processing on implicit surfaces and in level set methods. The existing data-extrapolating techniques are inefficient because they are designed without concerning the specialities of the extrapolating equations. Besides, there exists little work on locating the narrow band after data extrapolating—a very important problem in narrow band level set methods. In this paper, we put forward the general Huygens' principle, and based on the principle we present two efficient data-extrapolating algorithms. The algorithms can easily locate the narrow band in data extrapolating. Furthermore, we propose a prediction–correction version for the data-extrapolating algorithms and the corresponding band locating method for a special case where the direct band locating method is hard to apply. Experiments demonstrate the efficiency of our algorithms and the convenience of the band locating method.

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Keywords: Data extrapolating; Huygens' principle; Image processing on surfaces; Level set methods

1. Introduction

Data extrapolating is an indispensable step in image processing on implicit surfaces and in level set methods. Processing images on surfaces is considered to be a very difficult problem by classical methods. The PDE-based methods developed in recent years spark a minor revolution in image processing and have become the principle methods [12,13,3,2,16,18] for processing images on surfaces. Implicit surfaces are a popular type of surfaces with enough flexibility in geometric modelling. Hence, processing images on implicit surfaces is an interesting and important problem. There exists a little work on processing images on implicit surfaces. Osher et al. studied the problem of denoising images on implicit surfaces [2], and the authors of the present paper proposed an inpainting algorithm for images on implicit surfaces [18]. The basis approach of these work is to set up some energy functionals, and by minimizing the energy functionals some PDEs are derived. The PDEs are then solved numerically in a narrow band near the given implicit surface, where image data extrapolating is needed.

Data extrapolating also plays an important role in level set methods [5,7,10]. In level set methods, one needs to know the velocities in the neighborhood of the interface (zero level set) in order to move the interface. In some applications such as mean curvature flow and constant normal flow, the velocities are given globally or at least near the interface.

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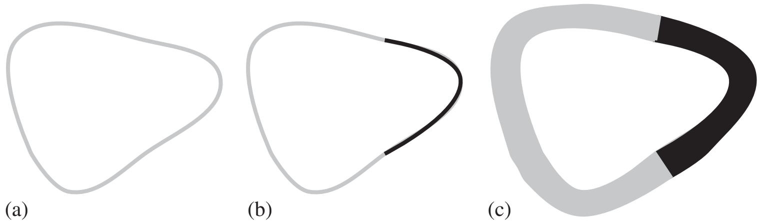

Fig. 1. Data extrapolating for an implicit curve: (a) an implicit curve, (b) data defined on the curve, and (c) the data are extrapolated in a band.

However, in most other applications such as the Stefan problem [5] and the Hele–Shaw flow problems [7], the velocities are given only at the interface. In these cases, one needs to extend the velocity to a band around the interface. After the data are extrapolated, one can then solve the corresponding level set equations in the band [10,15,4,6].

Data-extrapolating techniques are based on a Hamilton equation [10], which is also called the data-extrapolating equation. Assume an implicit surface S as the zero level set of φ is given. According to the data-extrapolating equation, data defined on S will be extended along the normals (outside of S) or the opposites (inside of S) of the level sets of φ. Fig. 1 illustrates an example of data extrapolating for an implicit curve. Here, a planar implicit curve is given in (a); in (b) a function (data) is defined on the curve, whose value is represented in gray on the left half while in black on the right half of the curve; (c) is the result of data extrapolating. The data defined on the curve are extrapolated to a band around the curve and the extrapolated data keep unchanged along the normal directions of the level sets of φ. Within the band around the curve as shown in Fig. 1(c), one can numerically solve the image processing equations or level set equations.

As far as the authors are aware, only a few data-extrapolating techniques are proposed so far. The first one is directly numerically solving the time-dependent data-extrapolating equation. According to its physical feature, a numerical method based on the upwind scheme can be adopted directly, see [10]. This method is simple in programming yet inefficient. If the data are extrapolated to the whole space, the algorithm complexity is $\mathrm{O}(N\sqrt[3]{N})$, where N is the number of grids in space. Peng [11] proposed a fast local level set method where the equation is solved only in a narrow band near the interface. In each time step, the numerical method in [10] is directly adopted and all the grids are visited and processed. The algorithm is also simple but with an algorithm complexity $\mathrm{O}(N)$ since the data are extrapolated in a narrow band near the interface. Unfortunately, it is not reported on how to locate the narrow band in detail, and the band locating method seems to be useful only for signed distance implicit functions.

The second class of methods is just considering the steady state of the data-extrapolating equation and solving a corresponding time-independent equation. Adalsteinsson et al. [1] proposed a global data-extrapolating method based on heap-sorting. This method is similar to the fast marching method [14] and has a lower algorithm complexity $\mathrm{O}(N \log N)$. But one should carefully design some complex data structures and implement a heap-sorting algorithm. As the steady-state equation of data extrapolating is a static Hamilton equation, fast sweeping methods based on Gauss–Seidel iteration can be adopted directly [9,17,8]. The algorithm complexity based on fast sweeping methods is $\mathrm{O}(N)$. However, it is a global method and cannot be localized easily.

In this paper, we will present two new data-extrapolating algorithms based on the general Huygens' principle. The algorithms make full use of the physical properties of the time-dependent data-extrapolating equation. By our methods, the data are extrapolated to the whole space with only a complexity $\mathrm{O}(N)$, and even with a lower complexity $\mathrm{O}(N^{2/3})$ if the data are extrapolated to only a narrow band. Our methods directly use the upwind scheme in each time step and do not involve any complex data structures. Thus, they are simple in programming. Unlike the local method in [11], only a small part of the grids needs to be processed in each time step by our methods. Furthermore, we can conveniently locate the narrow band for any implicit function according to the number of time steps in solving the data-extrapolating equation.

The paper is organized as follows. In Section 2, we analyze the data-extrapolating equation and present a method to locate the narrow band. The traditional data-extrapolating algorithm in [10,11] (for local case) is described briefly in Section 3. In Section 4, the general Huygens' principle is presented, and two fast data-extrapolating algorithms are proposed. In Section 5, for a special case where the locating of the narrow band is difficult, we put forward the prediction–correction version of the fast data-extrapolating methods. Experiments are provided in Section 6. Section 7 concludes the paper with a summary and future work.