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New error estimates for Galerkin method to an airfoil equation

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Abstract

In this paper we consider new error estimates under the weighted Chebyshev norm for Galerkin method to an airfoil equation. Galerkin discretisation is discussed. The smoothness conditions of the input functions are improved, i.e., they need to be Hölder continuous with $\frac{1}{2} < \mu < 1$.

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1. Introduction

In this paper we will discuss Galerkin solution of the following equation:

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\varphi(t)}{t-x}\,\mathrm{d}t+\lambda\int_{-1}^{1}k(t,x)\varphi(t)\,\mathrm{d}t=f(x)\quad(-1<x<1),\tag{1.1}$$

where $f(x)\in H^{\mu}[-1,1]$ is a real Hölder function on $[-1,1]$, $k(t,x)\in H^{\mu,\mu}[-1,1]\times[-1,1]$ is Hölder continuous on $[-1,1]\times[-1,1]$, λ is a suitable constant which is not the eigenvalue of (1.1). The singular integral in (1.1) is defined by

$$\int_{-1}^{1}\frac{\varphi(t)}{t-x}\,\mathrm{d}t=\lim_{\varepsilon\to 0}\left(\int_{-1}^{x-\varepsilon}+\int_{x+\varepsilon}^{1}\right)\frac{\varphi(t)}{t-x}\,\mathrm{d}t\quad(-1<x<1).\tag{1.2}$$

The solution $\varphi(x)$ belongs to one of the following function classes (see [15]):

$$h_0=\{\varphi(x):\varphi(x)\text{ is continuous in }(-1,1)\text{ and may have weak singularities both at }x=-1\text{ and }x=1\},$$

$$h(1)=\{\varphi(x):\varphi(x)\text{ is continuous in }(-1,1),\text{ bounded at }x=1\text{ and may have weak singularity at }x=-1\},$$

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$$h(-1) = \{\varphi(x) : \varphi(x) \text{ is continuous in } (-1, 1), \\ \text{bounded at } x = -1 \text{ and may have weak singularity at } x = 1\},$$

$$h(-1, 1) = \{\varphi(x) : \varphi(x) \text{ is continuous in } (-1, 1) \\ \text{and bounded both at the points } x = \pm 1\}.$$

These four function classes have the following relations:

$$h(-1, 1) \subset h(c) \subset h_0, \quad c = -1 \text{ or } 1. \tag{1.3}$$

According to the basic theory of singular integral equations [15], one of the function classes h_0, $h(-1)$, $h(1)$, $h(-1, 1)$ should be chosen as the solution class of $\varphi(x)$ when solving (1.1). The index and the canonical function can then be calculated with respect to this solution class. For example, consider the case where $\varphi(x)$ belongs to h_0. By the standard normalisation process for the singular integral equation (1.1), see [2,3,15], the index is $\kappa = 1$ and (1.1) is equal to the equations:

$$\begin{cases} \dfrac{1}{\pi} \int_{-1}^{1} \dfrac{u(t)}{\sqrt{1-t^2}(t-x)} \, dt + \lambda \int_{-1}^{1} (1-t^2)^{-1/2} k(t, x) u(t) \, dt = f(x), \\ \dfrac{1}{\pi} \int_{-1}^{1} \dfrac{u(t)}{\sqrt{1-t^2}} \, dt = C, \end{cases} \tag{1.4}$$

with $u(x)$ the Hölder continuous solution on $[-1, 1]$, and C an arbitrary constant. The relation between the solution $\varphi(x) \in h_0$ and $u(x)$, i.e., $\varphi(x) = (1 - x^2)^{-1/2} u(x)$, is a key point to discuss. Usually, it is difficult to approximate an unbounded function in h_0 directly, but it is much easier to discuss its equivalent equations (1.4) with Hölder continuous solution $u(x)$. Moreover, it connects the weight function $(1 - x^2)^{-1/2}$ with Chebyshev polynomials of the first kind. For the various numerical methods of (1.4), the best results on error estimates under the weighted L^2 norm and Chebyshev norm have already be obtained by Linz, Golberg, Ioakimidis and others ([14,4,12], etc.), so Galerkin solution of Eqs. (1.4) has been solved completely.

In this paper we consider (1.1) with $\varphi(x) \in h(-1)$. Note that the results will be analogous for $\varphi(x) \in h(1)$. Let

$$w(t) = \sqrt{\frac{1+t}{1-t}}, \quad w^{-1}(t) = \sqrt{\frac{1-t}{1+t}}, \tag{1.5}$$

the solution $\varphi(x)$ can be decomposed to $\varphi(x) = w(x)u(x)$. By the normalisation process of (1.1), the index $\kappa = 0$, and (1.1) is equivalent to

$$(\mathbf{A}u)(x) + \lambda(\mathbf{K}u)(x) = f(x) \quad (-1 < x < 1), \tag{1.6}$$

where

$$(\mathbf{A}\phi)(x) = \frac{1}{\pi} \int_{-1}^{1} \sqrt{\frac{1+t}{1-t}} \frac{\phi(t)}{t-x} \, dt,$$

$$(\mathbf{B}\phi)(x) = \frac{1}{\pi} \int_{-1}^{1} \sqrt{\frac{1-t}{1+t}} \frac{\phi(t)}{t-x} \, dt,$$

$$(\mathbf{K}\phi)(x) = \int_{-1}^{1} \sqrt{\frac{1+t}{1-t}} k(t, x) \phi(t) \, dt. \tag{1.7}$$

Eq. (1.6) is called the airfoil equation. Determining whether the solution $u(x)$ is continuous on $[-1, 1]$, depends on the smoothness properties of $f(x)$ and $k(t, x)$. Many papers have already discussed Collocation and Galerkin method for Eq. (1.6). The error estimate under the weighted L^2 norm has already been obtained by Ioakimidis, Golberg [5] and others under the conditions $f(x) \in H^{\mu}[-1, 1]$ and $k(t, x) \in H^{\mu,\mu}[-1, 1] \times [-1, 1]$, $0 < \mu < 1$, which could not be improved any more. But the error estimate under Chebyshev norm required more restricting conditions, i.e., $d^2 f/dx^2 \in H^{\mu}[-1, 1]$ and $\partial^2 k/\partial x^2(t, x) \in H^{\mu,\mu}[-1, 1] \times [-1, 1]$ ([6,4,7,8,13], etc.). Obviously, these conditions