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Parameter-dependent robust stability of uncertain time-delay systems

Huijun Gao[a], Peng Shi[a,b,*], Junling Wang[c]

[a] *Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin 150001, China*
[b] *School of Technology, University of Glamorgan, Pontypridd, CF37 1DL, UK*
[c] *College of Nuclear Science and Technology, Harbin Engineering University, Nantong Street 145, Harbin 150001, China*

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Abstract

This paper presents a new delay-dependent and parameter-dependent robust stability criterion for linear continuous-time systems with polytopic parameter uncertainties and time-varying delay in the state. This criterion, expressed as a set of linear matrix inequalities, requires no matrix variable to be fixed for the entire uncertainty polytope, which produces a less conservative stability test result. Numerical examples are given to show the effectiveness of the proposed techniques.

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Keywords: Delay-dependence; Linear matrix inequality; Parameter-dependence; Robust stability; Time-delay systems

1. Introduction

Time-delay systems, also called systems with after-effect or dead-time, hereditary systems, equations with deviating argument or differential-difference equations, have been a popular and challenging research area for the last few decades. The main reason is that many processes include after-effect phenomena in their inner dynamics, and engineers need their models to behave more like the real process due to the ever-increasing expectations of dynamic performance. So far, a great amount of effort has been devoted to the time-delay systems, and many interesting and important results have been reported in the literature (see for instance, [1–3,8,10,12–15,17–20,22,24] and the references therein).

Stability of time-delay systems has been well recognized to be a fundamental problem due to its important role in analysis and synthesis of such systems. The reported stability results can be generally divided into two categories: delay-independent and delay-dependent conditions. Delay-independent stability condition does not take the delay size into consideration, and thus is often conservative for systems with small delays. Therefore, in recent years, much attention has been drawn to the development of delay-dependent stability conditions, and many important results have been reported in the literature (see for example, [7,9,23] and the references therein). These stability conditions have also been extended to time-delay systems with parameter uncertainties, either in norm-bounded or polytopic forms.

* Corresponding author. Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin 150001, China. Tel.: +86 451 86402350/3141; fax: +86 451 86418091.

E-mail addresses: hjgao@hit.edu.cn (H. Gao), pshi@glam.ac.uk (P. Shi).

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For systems with time delay in the state and parameter uncertainties residing in a polytope, an advanced research topic is to develop robust delay-dependent and parameter-dependent stability conditions. The notion of parameter-dependence is introduced in order to overcome the conservativeness of the quadratic stability condition which requires a fixed Lyapunov function for the entire uncertain domain [5]. Many attempts have been made in the past few years to realize the parameter-dependent idea for systems with polytopic uncertainties [16]. Very recently, the parameter-dependent idea was further extended to time-delay systems, and some less conservative robust stability conditions have been proposed [6,11,21]. It should be noted that the techniques used to realize the parameter-dependent idea for time-delay systems in these papers are quite similar to that in [16]. That is, by introducing one or more additional slack matrix variables (without any structural restriction), the product terms between the Lyapunov matrices (positive matrices) and system matrices are eliminated, thus the Lyapunov matrices are allowed to be different for different vertices of the polytope. However, it is worth noting that in such treatment, some of those additionally introduced slack matrix variables are still required to be fixed for different vertices of the polytope, which is a common feature in these reported results [6,11,21].

In this paper, we present a new robust delay-dependent and parameter-dependent stability condition for time-delay systems with polytopic uncertainties. The most significant distinction of this stability condition from existing ones lies in the fact that it does not require any matrix variable to be fixed for different vertices of the polytope. In addition, it is shown that the stability condition proposed in [11] is a special case of the one reported in this paper. Two numerical examples are provided to illustrate the less conservativeness of our robust stability criterion.

Notations: The notation used throughout the paper is fairly standard. The superscript “T” stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P>0$ means that P is real symmetric and positive definite; I and 0 represent identity matrix and zero matrix. In symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Main results

Consider the following continuous-time system Σ with a time-varying delay in the state:

$$\begin{aligned} \Sigma:\ & \dot{x}(t) = A_\lambda x(t) + B_\lambda x(t-d(t)), \\ & x(t)=\phi(t), \quad t\in[-\bar{d},0], \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, and A_λ, B_λ represent uncertain system matrices belonging to a given convex polytope $\mathscr{R}$, in the following form

$$[A_\lambda \;\; B_\lambda] = \sum_{i=1}^{s} \lambda_i [A_i \;\; B_i], \tag{2}$$

where $\lambda \triangleq [\lambda_1 \;\; \lambda_2 \;\; \cdots \;\; \lambda_s]^{\mathrm{T}}$ denotes an uncertain vector satisfying

$$\sum_{i=1}^{s} \lambda_i = 1, \quad \lambda_i \geqslant 0 \tag{3}$$

and appropriately dimensioned matrix $[A_i \;\; B_i]$ denotes the ith vertex of the polytope $\mathscr{R}$.

In this paper, two cases of time-varying delay $d(t)$ will be considered

Case I:

$$0 < d(t) \leqslant \bar{d}, \quad \dot{d}(t) \leqslant \tau < 1, \tag{4}$$

Case II:

$$0 < d(t) \leqslant \bar{d}, \tag{5}$$

with $\bar{d}$ and τ being known positive constants. $\phi(t)$ is the initial condition on the segment $[-\bar{d},0]$.