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Specific locking in populations dynamics: Symmetry analysis for coupled heteroclinic cycles

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Abstract

Population dynamics on two sites of ecological fields are studied. Each site shows oscillatory dynamics with a heteroclinic cycle or a limit cycle attractor, and populations migrate between two sites diffusively. In this system, frequency locking states with specific ratios between the oscillations of two sites are observed. The selection of the ratios are explained with the symmetry of the phase space. Other properties of the locking states as behaviors intrinsic to heteroclinic cycles are also discussed.

MSC: 34C15; 34C14

Keywords: Coupled oscillators; Heteroclinic cycle; Frequency locking; Symmetry

1. Introduction

In models of ecological population dynamics, such as Lotka–Volterra equations or replicator systems, heteroclinic cycle or network attractors frequently appear [1–3,8]. A heteroclinic cycle is a structure in a phase space, it is constructed with some saddle fixed points and their cyclic connecting orbits (heteroclinic orbits). In a ecological model, the boundary of the phase space which is represented by a subset of species assembly (i.e., some species not participating the assembly have zero population) is robust. Thus, a heteroclinic cycle in the boundary can also exist robustly and be an attracting set. However, the existence of an attracting heteroclinic cycle means that the system approaches recursively and closer to these saddle fixed points at which some species have zero population. Hence, the populations of the species oscillate from exponentially small number to the dominant order (see Fig. 1 bottom). Corresponding phenomena in real eco-systems do not continue eternally, extinction are expected in finite times of access for the fixed points [7]. Therefore, situations corresponding to heteroclinic cycles are thought to be observed only as transient behaviors in this sense, thus heteroclinic cycles are not important structures to discuss the long term behaviors of eco-systems, as far as applying these models.

However, these standard models noted above do not have the spatial expanse, they correspond to the eco-systems which can be applied to the mean field approximation. When the concerning space is broad, population densities cannot hold the uniformity, and spatial differences would arise. Besides, dynamics of a trajectory along with a heteroclinic cycle is oscillatory, the spatial differences can grow to spatial patterns. In such cases, the existence of heteroclinic cycles

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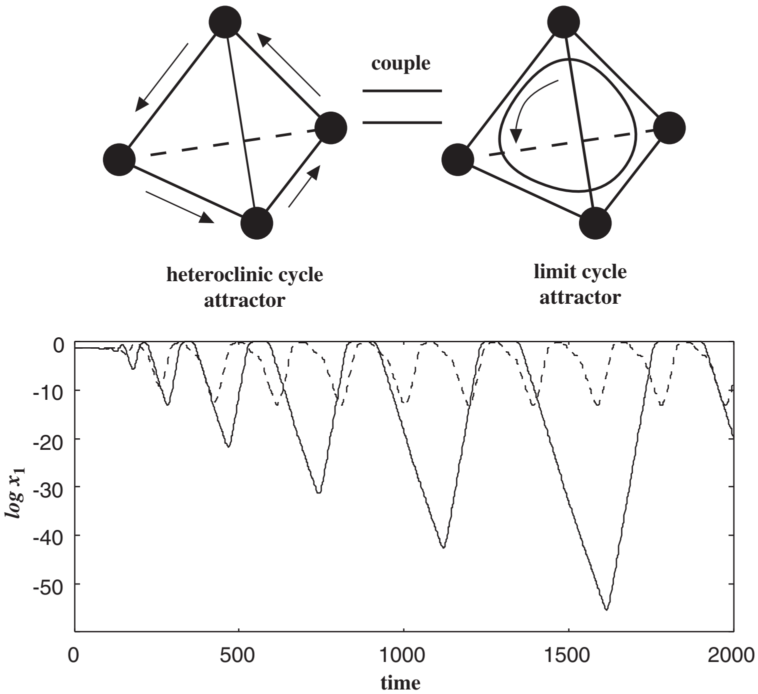

Fig. 1. Top figure: schematic diagram of coupled replicator systems with four species. The phase space of the replicator system in each site is boundary and interior of a tetrahedron, vertices of tetrahedron (closed circles) are saddle fixed points and their connecting edges (bold lines) are heteroclinic orbits. One replicator has a heteroclinic cycle attractor and the other has a limit cycle attractor in the vicinity of unstable heteroclinic cycle. Bottom figure: time series of $\log x_1$ in each site at the decoupled condition ($D = 0$). The replicator system with the limit cycle attractor (dashed line) show a bounded amplitude and a bounded period. On the other hand, in the replicator system with the heteroclinic cycle attractor (solid line) the period and the lowest value in each oscillation both diverge exponentially. In the normal scale the lowest value converges to zero.

in local dynamics brings only local extinction, and advections of populations provide extinct species to revolve local dynamics constantly. In this way, systems with heteroclinic cycle attractors can exist stationary in model eco-systems. Then, asking the importance of heteroclinic cycles becomes meaningful.

Here, we introduce multi site eco-system models in which some or all sites have local population dynamics with heteroclinic cycle attractors, and search for the behaviors that are essentially connected with the existence of the heteroclinic cycles. In our previous studies, we have shown the pattern dynamics intrinsic to heteroclinic cycles in a two dimensional lattice of sites with identical heteroclinic cycles [5,6].

2. Models

In the present paper, we suppose two sites with different conditions, each of which has mean field ecological systems written by replicator equations and they are coupled diffusively. The constituting species of replicator equations are same in two sites, parameters of replicator equations are chosen to be slightly different. Both sites have heteroclinic cycles with the same structure, one of which is set to be an attractor and the other site has an unstable (saddle) heteroclinic cycle. The destabilization of a heteroclinic cycle by the change of parameters is called heteroclinic bifurcation [3], which involves the birth of an attracting limit cycle from the heteroclinic cycle. Thus, the system we treat is a kind of coupled oscillator systems in which a limit cycle oscillator and a heteroclinic cycle oscillator are coupled. A schematic diagram of the systems and the behavior of each system in uncoupled state are shown in Fig. 1. In system with the heteroclinic cycle attractor in uncoupled condition, the oscillation of each value shows exponential divergence in the period and the lowest value of $\log x_1$ in each oscillation. Though in a coupled condition, the advection from the autonomous oscillating site excites the site continuously, and it is prospective to acquire the oscillation with a bounded period (Fig. 2).