Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational and Applied Mathematics 200 (2007) 408–423
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
www.elsevier.com/locate/cam

Stability criteria for certain high odd order delay differential equations

Baruch Cahlon*, Darrell Schmidt

Department of Mathematics and Statistics, Oakland University, Rochester, MI 48309-4401, USA

Received 26 April 2005; received in revised form 2 January 2006

Abstract

In this paper we study the asymptotic stability of the zero solution of odd order linear delay differential equations of the form

$$y^{(2m+1)}(t)=\sum_{j=0}^{2m} a_j y^{(j)}(t)+\sum_{j=0}^{2m} b_j y^{(j)}(t-\tau),$$

where a_j and b_j are certain constants and $m \geqslant 1$. Here $\tau > 0$ is a constant delay. In proving our results we make use of Pontryagin's theory for quasi-polynomials.

MSC: 45E99; 34D99

Keywords: Asymptotic stability; Stability criteria; Sufficient conditions; Delay; Characteristic functions; Stability regions

1. Introduction

The aim of this paper is to study the asymptotic stability of the zero solution of the delay differential equation

$$y^{(2m+1)}(t)=\sum_{j=0}^{2m} a_j y^{(j)}(t)+\sum_{j=0}^{2m} b_j y^{(j)}(t-\tau), \tag{1.1}$$

where $\tau > 0$, a_j, and b_j are constants and $m \geqslant 1$. In the previous papers [6,7], we considered Eq. (1.1) with $m = 1$ which arose from a robotic model with damping and delay, and in [3] we considered (1.1) with $m = 0$ and $a_0,\ b_0$ complex. There are no practical stability criteria of the zero solution of (1.1) for $m > 1$. For study of asymptotic stability of restricted special cases of (1.1) with special values of m see [7,3,22,21,9]. For stability and oscillation of certain third and fourth order equations see [17,8,10,20,23]. See [19,20,11,13,14] for studies of systems that may shed light on (1.1). The study on systems does not, however, yield practical stability criteria of (1.1). For further study on asymptotic stability see [15,12]. It is clear that with $4m + 2$ independent parameters in (1.1) one cannot expect to get regions of stability. Our goal is to derive algorithmic type stability criteria.

* Corresponding author. Tel.: +1 248 370 3435; fax: +1 248 370 4184.
E-mail address: cahlon@oakland.edu (B. Cahlon).

0377-0427/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2006.01.015

Our view is that part of the jth derivative term of the equation

$$y^{(2m+1)}(t)=\sum_{j=0}^{2m} p_j y^{(j)}(t) \tag{1.2}$$

is delayed and the remaining part is not. Note that with $\tau=0$ the zero solution of (1.1) or (1.2) is asymptotically stable if and only if all the characteristic roots of a real polynomial

$$x^{2m+1}-p_{2m}x^{2m}-p_{2m-1}x^{2m-1}-p_{2m-2}x^{2m-2}-\cdots-p_0=0 \tag{1.3}$$

are in complex left half plane. Relative to (1.1), we view

$$p_j=a_j+b_j,\quad j=0,1,\ldots,2m. \tag{1.4}$$

By Hurwitz Criterion [16] all roots have negative real parts if and only if

$$\delta_j>0,\quad j=1,2,\ldots,2m+1, \tag{1.5}$$

where the δ_j are the following determinants:

$$\delta_1=-p_{2m},$$

$$\delta_2=\begin{vmatrix}-p_{2m} & -p_{2m-2}\\ 1 & -p_{2m-1}\end{vmatrix},$$

$$\delta_k=\begin{vmatrix}-p_{2m} & -p_{2m-2} & -p_{2m-4} & \cdots & -p_{2(m-k)+2}\\ 1 & -p_{2m-1} & -p_{2m-3} & \cdots & -p_{2(m-k)+3}\\ 0 & -p_{2m} & -p_{2m-2} & \cdots & -p_{2(m-k)+4}\\ 0 & 1 & -p_{2m-1} & \cdots & -p_{2(m-k)+5}\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \cdots & -p_{2m-k+1}\end{vmatrix},\quad k=3,\ldots,2m,$$

with $-p_{2m+1-j}=0$ for $j>2m+1$.

The authors have previously applied Pontryagins principles to various cases of delay equations: first order complex coefficients, systems, and second order (see [3,2,4,5]). Our results and approaches are somewhat different than those in the first order complex coefficients and systems cases. To some extent, we employ the same approach as for the second order cases, but we also obtain some strong simplifications. In this paper we make use of the methods developed in [7]. It appears to the authors that an extension to general even order cases will be quite different.

This paper is organized as follows. In Section 2, we present the tools used in our asymptotic stability analysis. In Section 3 we give our main results and some special cases. In Section 4 we present some examples.

2. Background

In this section, we identify the characteristic function of (1.1) in order to study the asymptotic stability of the zero solution. We also cite the main results of Pontryagin related to asymptotic stability [18] and the applications of Pontryagin's results [1, Sections 13.7– 13.9].

The characteristic function of (1.1) is given by

$$\widehat{H}(s)=s^{2m+1}-\sum_{j=0}^{2m}a_js^j-\sum_{j=0}^{2m}b_j\mathrm{e}^{-s\tau}s^j. \tag{2.1}$$

Multiplying (2.1) by $\mathrm{e}^{s\tau}$ yields

$$\mathrm{e}^{s\tau}\widehat{H}(s)=\mathrm{e}^{s\tau}s^{2m+1}-\sum_{j=0}^{2m}a_js^j\mathrm{e}^{s\tau}-\sum_{j=0}^{2m}b_js^j. \tag{2.2}$$