Available online at www.sciencedirect.com
SCIENCE DIRECT®

ELSEVIER

Journal of Computational and Applied Mathematics 194 (2006) 192–206

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.com/locate/cam

State feedback control of switched linear systems: An LMI approach

V.F. Montagner[a], V.J.S. Leite[b], R.C.L.F. Oliveira[a], P.L.D. Peres[a,*]

[a]*DT/FEEC/UNICAMP, CP 6101, 13081-970, Campinas, SP, Brazil*
[b]*UnED Divinópolis, CEFET-MG, R. Monte Santo, 319, 35502-036, Divinópolis, MG, Brazil*

Received 16 July 2004; received in revised form 24 June 2005

Abstract

This paper addresses the problem of state feedback control of continuous-time switched linear systems with arbitrary switching rules. A quadratic Lyapunov function with a common matrix is used to derive a stabilizing switching control strategy that guarantees: (i) the assignment of all the eigenvalues of each linear subsystem inside a chosen circle in the left-hand half of the complex plane; (ii) a minimum disturbance attenuation level for the closed-loop switched system. The proposed design conditions are given in terms of linear matrix inequalities that encompass previous results based on quadratic stability conditions with fixed control gains. Although the quadratic stability based on a fixed Lyapunov matrix has been widely used in robust control design, the use of this condition to provide a convex design method for switching feedback gains has not been fully investigated. Numerical examples show that the switching control strategy can cope with more stringent design specifications than the fixed gain strategy, being useful to improve the performance of this class of systems.

Keywords: Switched linear systems; Switched control; Quadratic Lyapunov function; Linear matrix inequalities; Pole location; $\mathscr{H}_\infty$ control; Continuous-time systems

* Corresponding author. Tel.: +55 19 3788 3759; fax: +55 19 3289 1395.
E-mail addresses: montagne@dt.fee.unicamp.br (V.F. Montagner), valter@div.cefetmg.br (V.J.S. Leite), ricfow@dt.fee.unicamp.br (R.C.L.F. Oliveira), peres@dt.fee.unicamp.br (P.L.D. Peres).

0377-0427/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.cam.2005.07.005

1. Introduction

Switched systems are a class of hybrid systems consisting of several subsystems (modes of operation) and a switching rule indicating the active subsystem at each instant of time [5,17]. Among the dynamical systems that exhibit switching behavior, it is possible to cite the electrical circuits with electronic switches that define the important family of power converters [13], biochemical processes [7] and all the systems subject to switching control laws [15,21–23]. It is worth mentioning that switching control strategies are important to improve the performance of some systems, to guarantee their robustness and also to implement some adaptive control schemes [9]. Note also that switched systems can be used to model linear systems subject to actuator failures, that is, systems that can abruptly change the number of control inputs [24]. In the context of switched systems with linear continuous-time subsystems, the issues of stability analysis and control have been studied in the last few years [10,19,20] and, as stated in [16], the basic problems are: (a) to find the conditions that guarantee the system stability (or find a control law that guarantees the system stabilizability or even the achievement of some performance index) under any switching rule; (b) to find the switching strategy that guarantees the system stability (or the system stabilizability or even the achievement of some performance index). These and other issues such as the controllability and the reachability problems [25,27] have been under investigation and much attention has been paid on the switched linear systems [24,29]. Undoubtedly, a systematic way to study the stability and control problems for this class of systems is provided by the use of Lyapunov functions.

Several important results on robust stability and filtering for linear systems have been obtained in the last decades from the use of a common quadratic Lyapunov function [1]. This approach is appealing because of its numerical simplicity and also by the possibility to express the tests in terms of linear matrix inequalities (LMIs), solved in polynomial time by interior point based algorithms [6]. Another interesting feature is that the fixed Lyapunov function can cope with arbitrarily fast time-varying systems, thus encompassing the switched systems. Concerning the stability of autonomous switched linear systems, [18] shows that a common quadratic Lyapunov function exists if all the matrices are asymptotically stable and commute pairwise. The quadratic stabilizability problem for continuous-time systems in polytopic domains has been solved in [2], where an LMI condition is used to determine a fixed state feedback gain, allowing to deal with time-varying uncertain parameters and thus being useful for switched systems as well. Extensions of this condition have provided several results in robust control and filter design (see [3] and references therein), including performance indexes such as the $\mathscr{H}_\infty$ norm, which are entirely applicable to the robust stabilizability of switched linear systems with arbitrary switching rules by means of fixed gains.

The main drawback of the quadratic stabilization by means of fixed gains is that the results can be conservative, that is, the switched system may be stabilizable through switching strategies but does not admit a fixed constant stabilizing feedback for all subsystems. Some recent results explore the use of switching control strategies based on quadratic Lyapunov functions, as in [28], by means of a state dependent switching strategy and in [11], where parameterized LMIs need to be solved to obtain a set of gains that stabilize the closed-loop system for a given switching function. Many results based on quadratic stability to design switched stabilizing control gains for switched linear systems take into account prior information on the switching function or on the state space partitions, which in fact, when available, can be useful for control synthesis. However, there are systems for which this previous information is not available and thus the control design must be carried out for arbitrary switching functions. It is also worth to mention that results based on quadratic Lyapunov functions for stability analysis and control design