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A combined mixed and discontinuous Galerkin method for compressible miscible displacement problem in porous media☆

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Abstract

In this paper, we present a numerical scheme for solving the coupled system of compressible miscible displacement problem in porous media. The flow equation is solved by the mixed finite element method, and the transport equation is approximated by a discontinuous Galerkin method. The scheme is continuous in time and a priori hp error estimates is presented.

MSC: 65M60; 65N30

Keywords: Compressible miscible displacement; Mixed finite element method; Discontinuous Galerkin method; hp Error estimates

1. Introduction

We consider the compressible miscible model problem, which is given by the following equations with boundary and initial conditions [16]:

$$\begin{cases} d(c)\dfrac{\partial p}{\partial t} + \nabla \cdot u = d(c)\dfrac{\partial p}{\partial t} - \nabla \cdot (a(c)\nabla p) = q, \quad (x,t) \in \Omega \times J, \\ \phi\dfrac{\partial c}{\partial t} + b(c)\dfrac{\partial p}{\partial t} + u \cdot \nabla c - \nabla \cdot (D\nabla c) = (\hat{c} - c)q, \quad (x,t) \in \Omega \times J, \\ u \cdot v = 0, \quad (x,t) \in \partial\Omega \times J, \\ D\nabla c \cdot v = 0, \quad (x,t) \in \partial\Omega \times J, \\ p(x,0) = p_0(x), \quad x \in \Omega, \\ c(x,0) = c_0(x), \quad x \in \Omega. \end{cases} \tag{1.1}$$

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Here Ω is a polygonal domain in R^d $(d=2,3)$, $J=(0,T]$. The fluid pressure is denoted by p, the Darcy velocity $u=-a(c)\nabla p$, $\phi=\phi(x)$ is the porosity, $c=c(x,t)$ is the solvent (volumetric) concentration, and q is the external volumetric flow rate. The permeability of the medium is denoted by $k(x)$, and $\mu(c)$ is the viscosity.

In model (1.1), we confine ourselves to a two component displacement problem just for clarity of presentation. However, the numerical methods that we shall introduce and analyze below can be applied to the n component model. The coefficients appearing in (1.1) can be stated as follows:

$$c=c_1=1-c_2,$$

$$a(c)=a(x,c)=k(x)\mu(c)^{-1},$$

$$b(c)=b(x,c)=\phi(x)c_1\left\{z_1-\sum_{j=1}^{2}z_jc_j\right\},$$

$$d(c)=d(x,c)=\phi(x)\sum_{j=1}^{2}z_jc_j,$$

where z_j is the "constant compressibility" factor for the jth component.

In problem (1.1), the matrix $D=D(x)=\phi(x)d_mI$, and the notation $\hat{c}$ denotes the specified c_w at sources $(q>0)$ and the resident concentration at sinks $(q<0)$. If we put $q^+=\max(q,0)$ and $q^-=\min(q,0)$, then $q=q^++q^-$. We assume that the flow rate q is smoothly distributed in order to imply that the solution of the problem is smooth.

For problem (1.1), we need the following hypotheses (H):

(1) The mixture viscosity $\mu(c)$ has positive lower and upper bounds, and its derivative is uniformly Lipschitz continuous.
(2) There exist positive constants k_*, k^*, ϕ_*, ϕ^*, D_*, D^*, b^*, d_* and d^* such that

$$0<k_*\leqslant k(x)\leqslant k^*,\quad 0<\phi_*\leqslant\phi(x)\leqslant\phi^*,\quad 0<D_*\leqslant D(x)\leqslant D^*,$$

$$|b(\kappa)|\leqslant b^*,\quad 0<d_*\leqslant d(\kappa)\leqslant d^*,\quad \kappa\in R^1.$$

(3) There are two positive constants K_1 and K_2 such that

$$|q|\leqslant K_1,\quad \left|\frac{\partial q}{\partial t}\right|\leqslant K_2.$$

We make a few remarks for (2) in hypotheses (H). First, in real computations, once an approximate solution C for c is obtained, then we truncate C to [0,1], i.e., we use $C^*=\min(\max(C,0),1)$ instead of C [32]. For the brevity of presentation, we simply use $\kappa\in R^1$ instead of $\kappa\in[0,1]$. Secondly, the above assumptions made for $b(c)$ and $d(c)$ are reasonable, as it is easy to check that $\min_j z_j\leqslant\sum_{j=1}^2 z_jc_j\leqslant\max_j z_j$ as $\sum_{j=1}^2 c_j=1$ and $c_j\geqslant 0$ $(j=1,2)$. Under the above assumptions, we know that $\partial a/\partial c$ is uniformly bounded and Lipschitz continuous with respect to c.

In this paper, we consider the numerical solutions for the above coupled equations. First, we consider the numerical methods for the flow equation. To obtain a velocity by differencing or differentiating the resulting pressure determined by standard finite difference and finite element method then multiplying it by the rough coefficient will result in a rough and inaccurate velocity which will reduce the accuracy of numerical simulation of the fluid flow in porous media [17]. Mixed finite element method has the advantages that both the pressure and the velocity can have the same optimal order of convergence, and this method has been widely used in the numerical simulation for porous media problems since the early period of 1980s [14,15]. The p approximation results using the Raviart–Thomas–N*é*d*é*lec spaces were given in [21,36], and the hp-version was presented in [2,20,22].

Now we turn to the approximation schemes for the concentration equation. Discontinuous Galerkin finite element methods (DGFEMs) have become very popular in the science and engineering community now. They were introduced in the early seventies in the last century for solving the neutron transport equation [26]. In the paper written by Cockburn et al. [11], a general survey and a historical review were provided. In 1998, Oden et al. [24] presented an extension of the discontinuous Galerkin method for diffusion problems. Rivi*é*re et al. discussed DG methods for elliptic problems