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Measuring the performances of decision-making units using interval efficiencies☆

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Abstract

Efficiency is a relative measure because it can be measured within different ranges. The traditional data envelopment analysis (DEA) measures the efficiencies of decision-making units (DMUs) within the range of less than or equal to one. The corresponding efficiencies are referred to as the best relative efficiencies, which measure the best performances of DMUs and determine an efficiency frontier. If the efficiencies are measured within the range of greater than or equal to one, then the worst relative efficiencies can be used to measure the worst performances of DMUs and determine an inefficiency frontier. In this paper, the efficiencies of DMUs are measured within the range of an interval, whose upper bound is set to one and the lower bound is determined through introducing a virtual anti-ideal DMU, whose performance is definitely inferior to any DMUs. The efficiencies turn out to be all intervals and are thus referred to as interval efficiencies, which combine the best and the worst relative efficiencies in a reasonable manner to give an overall measurement and assessment of the performances of DMUs. The new DEA model with the upper and lower bounds on efficiencies is referred to as bounded DEA model, which can incorporate decision maker (DM) or assessor's preference information on input and output weights. A Hurwicz criterion approach is introduced and utilized to compare and rank the interval efficiencies of DMUs and a numerical example is examined using the proposed bounded DEA model to show its potential application and validity.

Keywords: Data envelopment analysis; Anti-ideal DMU; Bounded DEA; Worst relative efficiency; Inefficiency frontier; Interval ranking

1. Introduction

Data envelopment analysis (DEA), developed in [2], measures the efficiencies of decision-making units (DMUs) within the range of less than or equal to one and evaluates their performances using the best relative efficiencies. If a DMU is evaluated to have the best relative efficiency of one, then it is said to be DEA efficient; otherwise it is non-DEA efficient. DEA efficient DMUs are usually thought to perform better than non-DEA efficient DMUs.

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However, if efficiencies are measured within the range of greater than or equal to one, then the worst relative efficiencies can be used to measure the worst performances of DMUs. Contrary to the best relative efficiencies, which determine an efficiency frontier, the worst relative efficiencies of DMUs define an inefficiency frontier. If a DMU is evaluated to have the worst relative efficiency of one, then it is said to be DEA inefficient; otherwise, it is non-DEA inefficient. DEA inefficient DMUs are usually thought to perform worse than non-DEA inefficient DMUs.

From the above analyses we can see that efficiency is a relative measure. It can be measured either within the range of less than or equal to one, or within the range of greater than or equal to one. When measured within different ranges, it has different meanings. The resultant assessment conclusions are usually different. Any assessment using only one type of efficiency is obviously one-sided. Ideally, both types of efficiencies should be used at the same time to assess the performances of DMUs.

Doyle et al. [5] and Entani et al. [6] are the few persons, to the best of our knowledge, to consider and measure efficiencies from both the optimistic and the pessimistic points of view. Their models have similar structures and all have some significant drawbacks, which will be seen very clearly in our later discussion in Section 3.1.

In this paper, we reconsider the problem of performance measurement. We measure the efficiencies of DMUs within the range of an interval so that the best and the worst relative efficiencies can be measured within a unified DEA model framework. In order to determine the range of interval efficiency, a virtual anti-ideal DMU is introduced, whose performance is definitely the worst among all the DMUs. So, its best relative efficiency can be utilized as the constraint on the lower bound efficiencies of DMUs. A new DEA model with the upper and lower bounds on efficiencies is thus developed to compute the best and the worst relative efficiencies of each DMU, which constitute an interval to give an overall measurement and assessment of the performance of each DMU. Since the performances of DMUs are characterized by interval efficiencies, an appropriate ranking approach for interval numbers is needed.

The rest of the paper is organized as follows: in Section 2, we briefly introduce the basic DEA models for measuring the best and the worst relative efficiencies of DMUs. Section 3 analyses Entani et al.'s DEA models, points out their drawbacks and develops the bounded DEA model for crisp input and output data. Section 4 briefly introduces the Hurwicz criterion approach (HCA) for comparing and ranking interval efficiencies of DMUs. This is followed by a numerical example, which is provided to show the potential application of the proposed bounded DEA models. The paper is concluded in Section 6.

2. DEA models for measuring the best and the worst relative efficiencies of DMUs

2.1. CCR model for measuring the best relative efficiencies of DMUs

Assume that there are n DMUs to be evaluated, each DMU with m inputs and s outputs. We denote by $x_{ij}\,(i=1,\ldots,m)$ and $y_{rj}\,(r=1,\ldots,s)$ the values of inputs and outputs of $\mathrm{DMU}_j\,(j=1,\ldots,n)$, which are all known and positive. According to the implication of efficiency, the efficiency of DMU_j is defined as

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \tag{1}$$

where u_r and v_i are output and input weights assigned to the rth output and the ith input, respectively. In order to determine the efficiency of DMU_j relative to the other DMUs, Charnes et al. [2] developed the following well-known CCR model, which measures the best efficiencies of DMUs within the range of less than or equal to one:

$$\begin{aligned} \text{Max}\quad & \theta_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \\ \text{s.t.}\quad & \theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j=1,\ldots,n, \\ & u_r, v_i \geqslant \varepsilon, \quad r=1,\ldots,s; \quad i=1,\ldots,m, \end{aligned} \tag{2}$$

where the subscript zero represents the DMU under evaluation, u_r and v_i are decision variables and ε is the non-Archimedean infinitesimal. Through Charnes–Cooper's transformation [1], the fractional programming above can be