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On computer-assisted proofs for solutions of linear complementarity problems

Uwe Schäfer

Institut für Angewandte Mathematik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

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Abstract

In this paper we consider the linear complementarity problem where the components of the input data M and q are not exactly known but can be enclosed in intervals. We compare three tests to each other each of which can be used by a computer that supports interval arithmetic to give guaranteed bounds for a solution of the LCP defined by M and q.
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1. Introduction

Given $M \in \mathbb{R}^{n\times n}$ and $q \in \mathbb{R}^n$ the linear complementarity problem (LCP) is to find two vectors w, z such that

$$w - Mz = q, \quad w \geqslant o, \quad z \geqslant o, \quad w^{\mathrm{T}} z = 0, \tag{1}$$

or to show that no such vectors exist. Obviously, (1) is equivalent to

$$q + Mz \geqslant o, \quad z \geqslant o, \quad (q + Mz)^{\mathrm{T}} z = 0 \tag{2}$$

via $w := q + Mz$. The inequalities in (1), (2), and in the sequel are meant componentwise and o denotes the zero vector. The LCP models many important mathematical problems and there exist several algorithms for calculating numerical solutions of the LCP (see [4,5,9,12]). In [3,2], validation methods were presented that prove (by the use of a computer) guaranteed bounds on the distance between a numerical solution and an exact solution of the LCP. In the present paper we want to extend these ideas to the case that (due to representation errors, for instance) the components of M and q are not exactly known, but can be enclosed in intervals.

Example 1.1. In [14], the LCP arises with

$$M = \begin{pmatrix} W^{\mathrm{T}} A^{-1}(W + N_G) & I^{\mathrm{T}} \\ N_H - I & O \end{pmatrix}, \quad q = \begin{pmatrix} W^{\mathrm{T}} A^{-1} h + b \\ o \end{pmatrix}, \tag{3}$$

where A is assumed to be regular. Since, in general, $W^{\mathrm{T}} A^{-1}(W + N_G)$ and $W^{\mathrm{T}} A^{-1} h$ are not representable by floating point numbers, any algorithm applied on a computer without taking care of roundoff errors will, in general, not consider

E-mail address: Uwe.Schaefer@math.uni-karlsruhe.de.

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the original LCP defined by (3). However, a programming language that supports interval arithmetic will give us an interval matrix $[M]$ and an interval vector $[q]$ satisfying $M \in [M]$ and $q \in [q]$.

Recall that we consider compact intervals $[\underline{a}, \overline{a}] := \{x \in \mathbb{R} : \underline{a} \leqslant x \leqslant \overline{a}\}$ and denote the set of all such intervals by **IR**. We also write $[a]$ instead of $[\underline{a}, \overline{a}]$. Furthermore, we consider matrices with intervals as elements; i.e., $([a_{ij}]) = ([\underline{a}_{ij}, \overline{a}_{ij}])$. We also write $[\underline{A}, \overline{A}] := \{A \in \mathbb{R}^{n\times n} : \underline{A} \leqslant A \leqslant \overline{A}\}$. By $\mathbf{IR}^{n\times n}$ we denote the set of all these so-called interval matrices. We also write $[A]$ instead of $[\underline{A}, \overline{A}]$. The set of interval vectors with n components is constructed in the same way and is denoted by $\mathbf{IR}^n$. For an introduction to interval computations we refer to [1].

In the present paper we consider the case that there are given an interval matrix $[M] \in \mathbf{IR}^{n\times n}$ and an interval vector $[q] \in \mathbf{IR}^n$, and we consider the LCP (1) and (2), respectively, with $M \in [M]$ and $q \in [q]$. In Section 2, we present an algorithm that calculates vectors $[w], [z] \in \mathbf{IR}^n$ attacking

$$\forall M \in [M], \quad \forall q \in [q] \exists w^* \in [w], \quad z^* \in [z] \quad \text{such that } w^*, z^* \text{ fulfill (1).} \tag{4}$$

In Section 3, on the other hand, we consider computational tests on a given interval vector $[z] \in \mathbf{IR}^n$ (sometimes also called test box) attacking

$$\forall M \in [M], \quad \forall q \in [q] \exists z^* \in [z] \quad \text{such that } z^* \text{fulfills (2).} \tag{5}$$

Finally, in Section 4, we will present some numerical examples.

2. The Lemke algorithm

Considering the LCP one has to mention the Lemke algorithm, "which remains the most versatile algorithm for solving this fundamental problem in the field of mathematical programming" [6]. Therefore, we have implemented an interval arithmetic version of the Lemke algorithm [16] in the same manner as the Gaussian algorithm was extended to the interval Gaussian algorithm [1]. If it terminates with two interval vectors $[\underline{w}, \overline{w}], [\underline{z}, \overline{z}] \in \mathbf{IR}^n$ satisfying $\underline{w} \geqslant o$ and $\underline{z} \geqslant o$, then $[w]$ and $[z]$ fulfill (4), see Theorem 4.1 in [16]. However, there arise problems when solutions are degenerate, see Example 2.1.

Example 2.1. We have considered Example 1.1 with $n = 24$ and randomly chosen A, W, I, N_H, N_G, h, and b. The interval Lemke algorithm implemented in PASCAL-XSC [8] gave $[w] = o$ and

$$[z] = \begin{pmatrix} [-4.6E-011, 4.6E-011] \\ \vdots \\ [-4.6E-011, 4.6E-011] \\ [9.7977195916E-001, 9.7977195921E-001] \\ [1.9595439183E+000, 1.9595439185E+000] \\ [2.9393158775E+000, 2.9393158776E+000] \\ [3.9190878366E+000, 3.9190878368E+000] \\ [4.8988597958E+000, 4.8988597960E+000] \\ [5.8786317550E+000, 5.8786317552E+000] \\ [6.8584037142E+000, 6.8584037144E+000] \\ [7.8381756734E+000, 7.8381756736E+000] \\ [8.81794763259E+000, 8.81794763271E+000] \\ [9.79771959177E+000, 9.79771959189E+000] \\ [1.07774915509E+001, 1.07774915511E+001] \\ [1.17572635101E+001, 1.17572635103E+001] \end{pmatrix} \in \mathbf{IR}^{24}.$$

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