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Nonlinear filtering for jump-diffusions

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Abstract

The paper treats the nonlinear filtering problem for jump-diffusion processes. The optimal filter is derived for a stochastic system where the dynamics of the signal variable is described by a jump-diffusion equation. The optimal filter is described by stochastic integral equations.

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1. Introduction

We are interested in the nonlinear filtering model of the form

$$x(t) = x_0 + \int_0^t b(x(s))\,\mathrm{d}s + \int_0^t \sigma(x(s))\,\mathrm{d}W(s) + J(t),$$

where b and σ are bounded and continuous functions on $\mathbb{R}$, W a standard Brownian motion and the jump process $J(.)$ defined with

$$J(t) = \int_0^t \int_\Gamma q(x(s-), \rho) N(\mathrm{d}s, \mathrm{d}\rho),$$

where N is a Poisson random measure and J is independent of W. The Poisson random measure enables us to identify the times and the magnitudes of jumps as points of a Poisson process. The process $x(t)$ is partially observed by the process $y(t)$. We will discuss separately two different cases of the observation process $y(t)$, the first one given by

$$\mathrm{d}y(t) = h(x(t))\,\mathrm{d}t + \mathrm{d}B(t)$$

for a bounded, continuous function h and a standard Brownian motion B, independent of x and the second one by

$$y(t) = N(t)$$

for the Poisson process $N(t)$ with rate that may depend on $x(t)$ and with disjoint jumps with $J(t)$. There are some works also for the case, when the state process is modelled as a jump process that has common jumps with the observations,

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see [1]. In that case the state variable and the observation process cannot be made independent under a transformation of measures, which was the method used in our paper.

The problem is to derive the least squares estimate of $f(x(t))$, given all the observations of y up to time t,

$$E[f(x(t))|\mathscr{Y}_t],$$

where $\mathscr{Y}_t = \sigma\{y(s), s \leqslant t\}$.

The nonlinear filtering problem for jump-diffusion is of great interest in stochastic systems theory. It is well known that, if any of the functions in the system or observation models are nonlinear or if a jump term is present, then it is rarely possible to obtain the conditional distribution by a "finite computation". Thus, it seems natural to look for numerical approximations. This was done by several authors, see for example Kushner [10] or Di Masi and Runggaldier [3,4].

Recently, in finance a nonlinear filtering problem with marked point process observations was considered by Frey and Runggaldier [6]. More precisely, they developed recursive methods to compute approximations to the conditional distribution of this state variable. Their results were extended by Cvitanić et al. [2] with the exact mean-square optimal filtering equations for the unobservable process that allow to compute explicitely some conditional distributions. We use the similar idea to derive the stochastic differential equations for our optimal filter in continuous time observations, using the measure transformation approach.

The models for the signal process and the observation processes explored in this paper are defined in Sections 2 and 3 where the measure transformation approach to derive the equations for the filter in both cases of the observations is described. In Section 4 the unnormalized filtering equations for both cases are derived, following the idea of Malcolm et al. in [13]. In Section 5 we give the normalized versions of those filtering equations. We conclude with an explicit application in Section 6.

2. Jump-diffusion processes

In this section we define two processes: one is the signal process $\{x(t), 0 \leqslant t\}$, and the other is the process of observations $\{y(t), 0 \leqslant t\}$, both defined on the same probability space $(\Omega, \mathscr{F}, P)$.

Let b and σ be bounded and continuous $\mathbb{R}$-valued functions on $\mathbb{R}$. For $\Gamma \subseteq \mathbb{R}$, let $q(.,.)$ be a bounded and measurable function on $\mathbb{R} \times \Gamma$, continuous in the first argument and uniformly continuous in the second.

The *jump-diffusion process* $x(t)$ is given by the stochastic equation

$$x(t) = x_0 + \int_0^t b(x(s))\,\mathrm{d}s + \int_0^t \sigma(x(s))\,\mathrm{d}W(s) + J(t), \tag{1}$$

where $\{W(t), 0 \leqslant t\}$ is a standard $\mathbb{R}$-valued Brownian motion and the jump term $J(.)$ has the representation

$$J(t) = \int_0^t \int_\Gamma q(x(s-), \rho)N(\mathrm{d}s, \mathrm{d}\rho). \tag{2}$$

In this equation N is a Poisson random measure on the Borel sets of $[0, \infty) \times \Gamma$, independent of the Brownian motion W, with mean rate $EN(t + \Delta, A) - EN(t, A) = \lambda\Delta\pi(A)$, where λ is a real number (the jump rate) and $\pi(.)$ is a probability measure on the space of jumps Γ. It is known that there is a unique weak-sense solution of (1) for each initial condition x_0. See [11] and the references therein.

The process of observations $y(t)$ is defined by

$$\mathrm{d}y(t) = h(x(t))\,\mathrm{d}t + \mathrm{d}B(t) \tag{3}$$

for a bounded and continuous function h and a standard Brownian motion B, which is independent of x.

Denote by $\mathscr{Y}_t = \sigma\{y(s), s \leqslant t\}$ the σ-field generated by the observations, by $\mathscr{F}_t = \sigma\{x(s), s \leqslant t\}$ the σ-field generated by the signal, and by $\mathscr{G}_t = \sigma\{x(s), y(s), s \leqslant t\}$ the global σ-field generated by the signal process and the observations.

Let $f \in C_0^2(\mathbb{R})$. The filtering problem of interest is the computation of the least squares estimate of $f(x(t))$ given the observation information $\mathscr{Y}_t$:

$$E[f(x(t))|\mathscr{Y}_t].$$