Physical Communication 1 (2008) 266–276

Contents lists available at ScienceDirect

Physical Communication

journal homepage: www.elsevier.com/locate/phycom

Full length article

About the diversity in cyclic prefixed single-carrier systems

Bertrand Devillers *, Jérôme Louveaux, Luc Vandendorpe

Communications and Remote Sensing Laboratory, Université catholique de Louvain, place du Levant 2, B-1348 Louvain-la-Neuve, Belgium

ARTICLE INFO

Keywords:
Mutli-carrier
OFDM
Single-carrier
Cyclic prefix
Diversity analysis

ABSTRACT

This paper analyzes to what extent the multipath diversity is extracted by a cyclic prefixed single-carrier (CPSC) transmission. The state-of-the-art is currently pessimistic on that issue, and has to be moderated: if it is true that the asymptotical, i.e. at infinite signal to noise ratio (SNR), diversity order achieved by CPSC is equal to one, we prove that the block size has an influence on the performance at moderate SNR. In particular, for reasonably large values of the block size, we show that the multipath diversity can be extracted by CPSC for the range of bit error rate values typically used in practice. The influence of suboptimal linear receivers on the diversity extraction is also investigated.

1. Introduction

Achieving reliable and high data rate communications over wireless links remains a challenging problem. One of the reasons for that is to be found in the nature of the wireless medium itself. Because of interactions with the elements constituting the wireless environment, the received signal is the sum of multiple delayed and attenuated versions of the transmitted signal. In the jargon, the wireless channel is referred to as multipath fading channel. At first, the multipath fading channel was seen as an imperfection which had to be compensated. However, attitudes have changed rapidly as people realized that reliable communication can be achieved if the intrinsic nature of the wireless channel is exploited. In fact, since multiple replicas of the transmitted signal reach the receiver, the communication can be potentially reliable if at least one of these replicas has sufficient strength. In short, the multipath fading channel inherently provides a source of diversity, referred to as multipath diversity.

A key issue in digital wireless communication was to find strategies for how to deal with multipath fading channels, that is how to mitigate the intersymbol interference inherent to such channels. One well-known approach is the multi-carrier (MC) modulation, whose first efficient implementations were proposed more than 40 years ago. Since then, the popularity of the MC modulation has grown significantly thanks to its implementation with fast Fourier transform (FFT) [1,2]. Nowadays, the MC modulation is widely used [3,4] and is known as discrete multitone (DMT) and orthogonal frequency division multiplexing (OFDM) in the wireline and wireless communities respectively. The basic idea relies on the organizing of the transmission in blocks of symbols and on the addition of a cyclic prefix (CP) (as first introduced in [1]) to each transmitted block. The role of the CP is twofold: first it acts as a guard period between blocks, and second it converts the linear convolution with the channel impulse response into a cyclic one. Thanks to that, the equalization can be linearly implemented in the frequency domain, with low complexity: one inverse FFT (IFFT) and one FFT are needed at the transmitter and receiver sides, respectively.

More recently, it has been pointed out in [5] that by keeping the cyclic extension of each block and by moving the IFFT operation of the OFDM modulation from the transmitter to the receiver side, we end up with a so-called cyclic prefixed single carrier (CPSC) system. The CPSC system also benefits from a low complexity equalization of the multipath channel in the frequency domain [6]. The complexity is actually mostly moved to the receiver side.

* Corresponding author.
E-mail addresses: bertrand.devillers@uclouvain.be (B. Devillers), jerome.louveaux@uclouvain.be (J. Louveaux), luc.vandendorpe@uclouvain.be (L. Vandendorpe).

1874-4907/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.phycom.2008.12.001

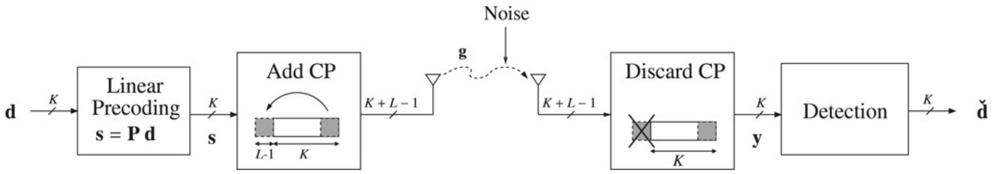

Fig. 1. Linearly precoded cyclic prefixed block transmission: block diagram.

On top of that, CPSC transmission has received much attention because, unlike the OFDM modulation, it does not suffer from the peak to average power ratio (PAPR) problem. This is precisely due to the absence of the IFFT operation at the transmitter side. All these considerations make CPSC a good alternative candidate for the uplink [7].

The OFDM and CPSC schemes have been extensively compared [5–10], especially from a system-based point of view. However, quite surprisingly, there have been very few comparisons based on analytical results [11, 12]. In particular, one might be interested in analyzing to what extent the multipath diversity is extracted by these schemes. In [13], it was proved that zero-padding single-carrier transmission naturally exploits the full diversity. The state-of-the-art is currently much more pessimistic for CPSC: the authors in [14,15] proved that, strictly speaking, CPSC is unable to extract any multipath diversity even with maximum likelihood reception. In this paper, we do not question this result but still moderate it as we prove that the multipath diversity is still exploited by CPSC under some realistic hypothesis. Moreover, in this context, the influence of suboptimal linear receivers is also investigated.

The rest of the paper is organized as follows. The notion of cyclic prefixed block transmission is introduced in Section 2. The associated signal model is also derived. It is shown how this signal model takes the shape of either an OFDM or CPSC system depending on the linear operation implemented at the transmitter side. In Section 3, the OFDM and CPSC schemes are compared in terms of instantaneous BER, i.e. for a given channel realization. Section 4 goes one step further and takes into account the fading of the wireless channel, which leads to the core of this paper: the derivation of diversity issues. Finally, conclusions are given in Section 5.

2. Cyclic prefixed block transmission: From multi-carrier to single-carrier

Cyclic prefixed block transmission has become a well-known approach for dealing with multipath channels with low complexity. The role of the cyclic prefix (CP) is twofold. Firstly, it acts as a guard period preventing interference between successive blocks. Secondly, the CP converts the linear convolution with the channel impulse response into a cyclic one. To play efficiently these two roles, the CP must be of length greater or equal to the channel impulse response's delay, i.e. equal to $L-1$ symbol periods if the discrete channel impulse response has L taps

$$\mathbf{g} = [g_0, g_1, \ldots, g_{L-1}]^{\mathrm{T}}. \tag{1}$$

With adequate CP inserting and discarding at the transmitter and receiver side respectively, the received $K \times 1$ vector $\mathbf{y}$ is then given by

$$\mathbf{y} = \mathbf{G}_c\, \mathbf{s} + \mathbf{n} \tag{2}$$

where K denotes the block size, $\mathbf{s}$ the $K \times 1$ transmitted vector, and $\mathbf{n}$ the additive white Gaussian noise vector. The matrix $\mathbf{G}_c$ is a $K \times K$ circulant matrix whose first column is given by the channel impulse response $\mathbf{g}$ appended by $K-L$ zeros. Let us introduce the K-point fast Fourier transform (FFT) of the channel impulse response

$$\boldsymbol{\omega} = [\omega_0, \omega_1, \ldots, \omega_{K-1}]^{\mathrm{T}} \tag{3}$$

$$\omega_k = \sum_{l=0}^{L-1} g_l \mathrm{e}^{-j2\pi \frac{k\,l}{K}}. \tag{4}$$

It is well-known that a circulant matrix can be decomposed as

$$\mathbf{G}_c = \mathbf{W}^H \boldsymbol{\Omega}\, \mathbf{W} \tag{5}$$

where $\boldsymbol{\Omega}$ is a diagonal matrix with the frequency channel response $\boldsymbol{\omega}$ on its diagonal, i.e. $\boldsymbol{\Omega} = \mathrm{diag}(\boldsymbol{\omega})$, and $\mathbf{W}$ (resp. $\mathbf{W}^H$) is the $K \times K$ FFT (resp. IFFT) unitary matrix.[1] In other words, the channel matrix $\mathbf{G}_c$ in the signal model (2) has always the same eigenvectors, independently of the realization of $\mathbf{g}$. This is precisely the convenient property we get thanks to the CP.

In this paper, we will assume that the noise samples n_m and channel taps g_l can be modeled as i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variances σ_n^2 and $1/L$, respectively. Note that this normalization is such that $\mathbf{g}$ has unit average power, independently of the value of L. Consequently, looking at the squared norm of a given tap, we end up with a random variable $x = |g_l|^2$ which has a chi-square distribution. Its probability density function (pdf) is given by

$$T_x(x) = L\mathrm{e}^{-Lx}. \tag{6}$$

Let us then consider the linearly precoded cyclic prefixed block transmission depicted in Fig. 1. This scheme assumes that the transmitted vector $\mathbf{s}$ is a linearly precoded version of the $K \times 1$ data symbol vector $\mathbf{d} = [d_0, d_1, \ldots, d_{K-1}]^{\mathrm{T}}$

$$\mathbf{s} = \mathbf{P}\, \mathbf{d}. \tag{7}$$

Throughout this paper, and without loss of generality, the data symbols d_k will be taken from a BPSK constellation. The detection operation was also included in Fig. 1. By

[1] The FFT matrix $\mathbf{W}$ is defined by $[\mathbf{W}]_{k,k'} = \frac{1}{\sqrt{K}} \mathrm{e}^{-j2\pi \frac{k\,k'}{K}}$.