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Theoretical framework of an identification problem for an elliptic variational inequality with bilateral restrictions

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Abstract

An identification problem associated to an elliptic variational inequation subject to a bilateral restriction is considered. The whole of the parameters involved in the inequation as well as the parameters defining the restriction are to be identified. The continuous dependence of the direct problem solution on these parameters is proved. As a consequence the well-posedness of the identification problem follows.

Keywords: Bilinear form; Elliptic variational inequalitiy; Bilateral restriction; Inverse problem; Coefficients identification

1. Introduction

It is well known that there are physical and engineering processes mathematically modeled by variational inequalities (VI) [6,3, p. 18]. Since the works of Lions and Stampacchia [16] up today a great deal of research has been developed on them and a general variational inequalities theory is at our disposal [18] to attack different applied problems. The VI approach is a fruitful way to analyze systems described by PDEs, mainly when there are state constraints or connections involving unilateral or bilateral restrictions. It can be seen in [15] where several cases are modeled and analyzed with the VI method. Following this line several papers deal with the numerical computation of the state of a coupled system described by PDEs [17,9,20,8]. Besides, Lions formulation [15] turns out to be useful for certain applications on aeronautics. Due to these applications the necessity of recovering the distributed coefficients involved in the mathematical model by using experimentally measured data appears. Hence an inverse problem associated to variational inequalities under unilateral or bilateral restrictions naturally arises. As usual the inverse problem is not well-posed and instead of it an identification problem must be formulated. The purpose of this work is to study the feasibility of an identification problem for the kind of problems previously referred to.

The problem of determining unknown coefficients for linear elliptic equations has extensively dealt with ([2] and references in it). There are also some antecedents about the same problem associated to variational inequalities ([10,11] and references within).

In this work, the direct problem (D.P.) consists of an elliptic variational inequation subject to a bilateral restriction. The whole of the parameters involved in the inequation as well as the parameters defining the restriction are to be

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identified. So stated the problem is quite similar to an identification problem (I.P.) for an elliptic equation. Indeed the development of this work follows analogous steps as in [2,13]. However, the fact that there are also unknown parameters in the associated restriction will require additional considerations.

Let us denote as q the vector of parameters which defines (D.P.) and $u = u(q)$ its (unique) solution. The identification problem (I.P.) is stated as a minimization criteria of an adequate functional $J(q)$ which involves the available measured data and the computed $u(q)$ for $q \in Q_0$ (the set of admissible coefficients). This work will establish the well-posedness of (I.P.), i.e. the existence of its solution. The resulting framework will also be useful for the numerical–computational implementation of the problem in future.

This work is organized as follows. In Section 2 the statement of the problem is presented. Based on previous well known results, a necessary topological framework is stated. In Section 3, the continuous dependence of the solution of (D.P.) on the parameter q is shown and so the well-posedness of (I.P.) is achieved.

2. General statement and standard results

2.1. The direct problem

The direct problem is given by

$$\begin{cases} a(u, v-u) \geqslant L(v-u) & \forall\, v \in K, \\ u \in K \end{cases} \tag{D.P.}$$

being $a(u, v) : H_0^1(\Omega) \times H_0^1(\Omega) \to \mathbb{R}$ a bilinear form defined as

$$a(u, v) = \int_\Omega \left(\sum_{i,j=1}^{n} a_{ij} u_{x_i} v_{x_j} + \sum_{i=1}^{n} b_i u_{x_i} v + cuv \right) \tag{1}$$

such that

$$a_{ij}, b_i, c \in L^\infty(\Omega) \quad 1 \leqslant i, j \leqslant n, \tag{2}$$

$$L(v-u) = (f, v-u), \tag{3}$$

where (., .) denotes the usual inner product in L^2 and

$$f \in L^2(\Omega), \tag{4}$$

$$K = \{v \in H_0^1(\Omega) : m(x) \leqslant v(x) \leqslant M(x) \text{ a.e. in } \Omega\} \tag{5}$$

with

$$m, M \in H^2(\Omega) \cap H_0^1(\Omega), \tag{6}$$

$$0 \leqslant m(x) \leqslant M(x) \leqslant 1 \text{ a.e. in } \Omega. \tag{7}$$

Concerning the domain Ω, throughout the work the following assumption holds.

Assumption. Ω is a bounded (open) set of $\mathbb{R}^n$.

In order to guarantee existence and solution of the direct problem additional hypothesis must be added on the parameters:

$$|a_{ij}(x)| \leqslant D_a, \quad |b_i(x)| \leqslant D_b, \quad 1 \leqslant i \leqslant j \leqslant n, \quad 0 < D_c^i \leqslant c(x) \leqslant D_c^s \quad \text{a.e. in } \Omega \tag{8}$$

with D_a, D_b, D_c^i, D_c^s positive constants.