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A variant of SQP method for inequality constrained optimization and its global convergence☆

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Abstract

In this paper, a variant of SQP method for solving inequality constrained optimization is presented. This method uses a modified QP subproblem to generate a descent direction as each iteration and can overcome the possible difficulties that the QP subproblem of the standard SQP method is inconsistency. Furthermore, the method can start with an infeasible initial point. Under mild conditions, we prove that the algorithm either terminates as KKT point within finite steps or generates an infinite sequence whose accumulation point is a KKT point or satisfies certain first-order necessary condition. Finally, preliminary numerical results are reported.

Keywords: SQP method; Inequality constrained optimization; Global convergence

1. Introduction

In this paper, we consider the nonlinear inequality constrained optimization problem

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & c_i(x) \leqslant 0, \quad i \in I, \end{aligned} \tag{1}$$

where $I = \{1, \ldots, m\}$, $f : R^n \to R$ and $c_i : R^n \to R$, $i \in I$, are continuously differentiable functions.

The method of sequence quadratic programming (SQP) is an important method for solving problem (1). At each iteration, the standard SQP method generates a decent direction by solving the following quadratic programming subproblem

$$\begin{aligned} \min \quad & \nabla f(x_k)^{\mathrm{T}} d + \tfrac{1}{2} d^{\mathrm{T}} B_k d \\ \text{s.t.} \quad & c_i(x_k) + \nabla c_i(x_k)^{\mathrm{T}} d \leqslant 0, \quad i \in I. \end{aligned} \tag{2}$$

where B_k is Hessian of Lagrangian function associated with (1). With an appropriate merit function, line search procedure, Hessian approximation procedure, and (if necessary) Maratos avoidance scheme, the SQP iteration is

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well-known to be globally and locally superlinearly convergent [8]. But the SQP method may fail if the linear constraints in quadratic programming subproblems (2) is inconsistency.

Many efforts have been made to overcome the difficulties associated with the inconsistency of quadratic programming subproblem (2). For example, Powell [10] suggested to solving a modified subproblem at each iterate:

$$\begin{aligned} \min \quad & \nabla f(x_k)^{\mathrm{T}} d + \tfrac{1}{2} d^{\mathrm{T}} B_k d + \tfrac{1}{2}\delta_k (1-\mu)^2 \\ \text{s.t.} \quad & \mu_i c_i(x_k) + \nabla c_i(x_k)^{\mathrm{T}} d \leqslant 0, \quad i \in I. \end{aligned}$$

where

$$\mu_i = \begin{cases} 1, & c_i(x_k) < 0 \\ \mu, & c_i(x_k) \geqslant 0 \end{cases} \quad \text{and} \quad 0 \leqslant \mu \leqslant 1, \quad \delta_k > 0$$

is a penalty parameter. The computational investigation provided by Schittkowski [12,13] showed that this modification worked very well. However, this approach may not be the best one for it cannot cope with a simple example presented by Burke and Han [4] and Burke [3].

Another approach was proposed by Burke and Han [4] and Burke [3]. Their methods can converge to a point which meets a certain first-order necessary optimality condition even when problem (1) is infeasible. Liu and Yuan [9] presented a method of the same convergent property with Burke and Han's. Their method solves two subproblems, one is an unconstrained piecewise quadratic subproblem, the other is a quadratic subproblem. In [17], Zhou presented a modified SQP method. Their method solves two subproblem, one is a linear programming with bound constraint, the other is a quadratic subproblem.

Recently, Zhang and Zhang [16] proposed a robust SQP method for solving problem (1). Similar to Zhou's method, at each iteration, their method solves a linear programming and a quadratic programming subproblem and is implementable. Under certain conditions, their method is globally convergence and locally superlinearly convergence.

In this paper, we describe another implementable method that can cope with the infeasibility of QP subproblem. Specifically, given $x_k \in R^n$, a symmetric positive definite matrix B_k, we solve a QP subproblem $QP(x_k; B_k)$ with the following form:

$$\begin{aligned} \min_{d,z} \quad & z + \tfrac{1}{2} d^{\mathrm{T}} B_k d \\ \text{s.t.} \quad & \nabla f(x_k)^{\mathrm{T}} d \leqslant z, \\ & c_i(x_k) + \nabla c_i(x_k)^{\mathrm{T}} d \leqslant z, \quad i \in I. \end{aligned} \tag{3}$$

Note that $QP(x_k, B_k)$ is always feasible for $d=0$ and $z = \max_{i \in I}\{c_i(x_k); 0\}$ satisfy the constraints of (3). Let (d_k, z_k) be the solution of $Q(x_k, B_k)$. If $d_k \neq 0$, then d_k is a decent direction of merit function. Under mild conditions, our algorithm is global convergent.

The QP subproblem which is similar to $QP(x_k, B_k)$ has recently been used in the constrained optimization by Birge et al. [2], Lawrence and Tits [8], Chen and Kostreva [5] and Kostreva [7]. They introduced the right-hand side constraint perturbation in (3) subproblem and used it to obtain a feasible direction. But, in this paper, our goal is to compute a descent direction even if the constraints in (2) is inconsistent.

The paper is organized as follows. Our algorithm is presented in Section 2. In Section 3, the global convergence results of the algorithm are proved. Some preliminary numerical results are reported in Section 4. Finally, the conclusions are given in Section 5.

2. The algorithm

In this section we define our SQP method for inequality constrained optimization. In our approach, the algorithm can start at any point $x \in R^n$.

In order to obtain the global convergence, we employ the penalty function associated with (1) as a merit function, i.e.,

$$\Psi_\sigma(x) = f(x) + \sigma\Phi(x),$$