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A hybrid linking approach for solving the conservation equations with an adaptive mesh refinement method

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Abstract

Accurate numerical computation of complex flows on a single grid requires very fine meshes to capture phenomena occurring at both large and small scales. The use of adaptive mesh refinement (AMR) methods, significantly reduces the involved computational time and memory. In the present article, a hybrid linking approach for solving the conservation equations with an AMR method is proposed. This method is essentially a coupling between the h-AMR and the multigrid methods. The efficiency of the present approach has been demonstrated by solving species and Navier–Stokes equations.

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1. Introduction

Accurate computational methods are required to understand complex phenomena occurring during incompressible turbulent and multiphase flows. It is essential to use a very fine mesh in order to catch small structures that appear locally at different space and time scales. In addition, in most multiphase configurations, the computational results may not be interpreted because of false numerical diffusion that may exceed the existing real physical diffusion. Vincent et al. [15] developed an original one-cell local multigrid (OCLM) method to solve the incompressible Naviers–Stokes equations for two-phase flows. This method is based on the concepts presented in [1,7] concerning an AMR method for compressible fluids. The equations are approximated by finite volumes on a marker and cell (MAC) grid [5]. This method consists in generating fine grids from a coarser one, by means of a gradient criterion. If a point M on the coarse level G_{l-1} verifies the criterion, the control volume around M is refined and a finer level G_l is built. The refined control volume is called AMR cell. An odd cutting is required to ensure a consistent connection between the fine grids of level G_l. The method can be explained by referring to Fig. 1 which shows a three-cutting of an AMR cell of level

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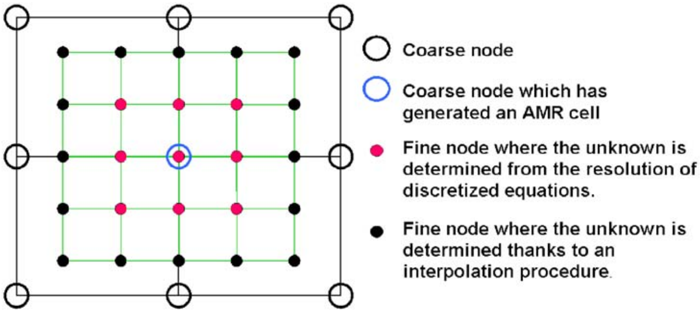

Fig. 1. AMR cell points treatment.

$$\int_{V_{father}} \Phi_{l-1} dvol \sim \sum_{i=1}^{9} \int_{V_{son(i)}} \Phi_{l-1} dvol.$$

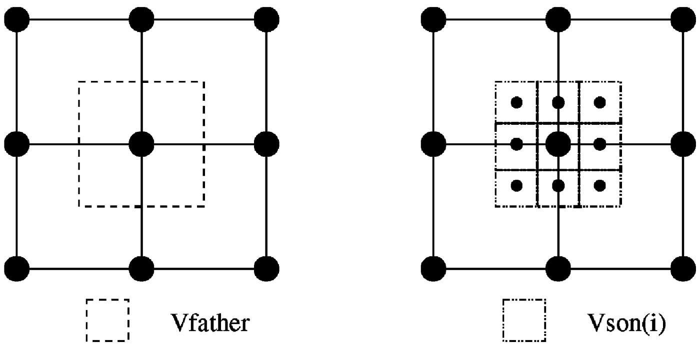

Fig. 2. Injection procedure.

G_l generated from a node of level G_{l-1}. When a third level G_{l+1} is generated, it is embedded in level G_l, which is itself embedded in level G_{l-1}. In fact, the OCLM method can be considered as a coupling between an h-AMR method (mesh enrichment [1,10]) and a multigrid method (calculation on several grids). The unknowns on points of level G_l generated by a point of level G_{l-1} are determined as explained below, see Fig. 1.

For the limits points of the AMR cell, the coarse solution is extrapolated to level G_l using a classical Q_1 interpolation procedure. For the interior points of the AMR cell, the conservation equations are solved on level G_l. Fig. 1 shows two kinds of nodes on level G_l: gray and black. Each gray node has all the neighbors needed to solve the discretized conservation equations on this node. But we note that the black nodes require neighbors outside it AMR cell. Therefore, a different approach is needed to solve them. An idea is to approximate the unknown on a black node by a polynomial of first, second or third degree. Then, the solution Φ_l, obtained on the nodes of the AMR cell is more accurate than the solution Φ_{l-1} on the node M_K^{l-1} which has generated it. As a result, a direct injection procedure or a full weighting interface control volume (FWICV) [4] is used to improve the accuracy of the solution Φ_{l-1} on the node M_K^{l-1}. It comes down to approximating the unknown Φ_{l-1} on level G_{l-1} by a more accurate solution Φ_l on level G_l as follows (Fig. 2):

$$\int_{V_{\mathrm{father}}} \Phi_{l-1}\,\mathrm{dvol} \sim \sum_{i=1}^{9} \int_{V_{\mathrm{son}(i)}} \Phi_{l-1}\,\mathrm{dvol}.$$

However, this method suffers from several weaknesses. First, level G_l is considered as a series of independent AMR cells for the solving of the conservation equations. In fact, the information does not go through cell to cell but from