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Asymptotic model for free surface flow of an electrically conducting fluid in a high-frequency magnetic field

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Dedicated to Roderick S.C. Wong on the occasion of his 60th birthday

Abstract

We study the behaviour of a layer of an electrically conducting inviscid incompressible fluid in a high-frequency alternating magnetic field. We derive nonlinear asymptotic equations governing the evolution of the fluid layer in the high-frequency limit. As a test for the model, we consider the linearised stability problem for an infinite planar free surface of a layer of finite depth.

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1. Introduction

Alternating electromagnetic fields are used in a number of metal processing techniques to levitate, stir and confine liquid metals. The effect of electromagnetic fields on the free surface of electrically conducting fluids has received considerable attention in the literature [2–5,7,8]. The simplest problem that has been studied by many authors (see, e.g., [2]) is the stability of a planar horizontal free surface of a conducting fluid in the presence of a horizontal alternating magnetic field. The field induces electric current in the

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fluid which, in turn, produces the Lorentz force acting on the fluid. At high frequency, the magnetic field penetrates only a shallow surface layer of the fluid. If we ignore finite thickness of this surface layer and regard it as a surface (of zero thickness), we arrive at the model where the fluid is perfectly conducting and where there is a current sheet at the free surface leading to an appropriate jump in tangent magnetic field. This idealized problem had been studied by Ladikov [4] who concluded that a high-frequency rotating magnetic field can stabilise a levitating layer of a liquid metal. Garnier and Moreau [3] had considered the linearised stability of a planar horizontal interface between a nonconducting fluid occupying half-space and carrying a uniform alternating magnetic field and an infinitely deep layer of a fluid of finite electric conductivity. They considered the limit of high frequency of the applied magnetic field and averaged the Lorentz force over the period. The result of their study demonstrated that the magnetic field is stabilising. Ramos and Castellanos [7] had taken into account viscosity of the fluid in a similar stability problem with a plane perfectly conducting cover at some distance from the interface. Parametric resonance at moderate frequency of the applied magnetic field has been studied by Cherepanov [1] in the case of perfectly conducting but viscous fluid and by Fautrelle and Sneyd [2] for a fluid with finite conductivity.

Most papers on the subject deal with the linearised stability problems in two limit cases: either the fluid is perfectly conducting or its conductivity is not only finite but large enough. Another approximation employed in many papers is that, while studying the linearised stability in high-frequency limit, an ad hoc procedure of averaging the Lorentz force is employed. Although the procedure seems to be physically reasonable, it is desirable to obtain the averaging procedure using some regular technique. It is the aim of the present paper to obtain the nonlinear equations that describe asymptotic behaviour of a layer of an inviscid fluid of finite electric conductivity in an arbitrary periodic magnetic field in the high-frequency limit.

The plan of the paper is as follows. In Section 2, we formulate the problem. In Section 3, we derive an asymptotic form of the governing equations using the method of multiple scales. Section 4 deals with the linearised stability of a planar free surface in the framework of the asymptotic model obtained in Section 3. Finally, in Section 5, we discuss the results.

2. Formulation of the problem

Consider a layer of a conducting inviscid fluid. The layer is bounded by a rigid perfectly conducting plane $z=-H$ and a free surface

$$z=\zeta(x,y,t) \tag{1}$$

and extends to infinity in x and y direction. In the space above the layer ($z>\zeta(x,y,t)$) there is vacuum (or a nonconducting gas of small density), and a periodic (in time) alternating magnetic field is applied at infinity (as $z\to\infty$). In the vacuum region, the magnetic field $\mathbf{B}(\mathbf{x},t)$ is irrotational

$$\mathbf{B}=\nabla\phi, \quad \nabla^2\phi=0 \quad \text{for } z>\zeta(x,y,t), \tag{2}$$

$$\nabla\phi\to\mathbf{B}^\infty(\omega t) \quad \text{as } z\to\infty, \tag{3}$$

where $\mathbf{B}^\infty(\omega t)$ is a periodic function of t with period $2\pi/\omega$ with zero mean value,

$$\overline{\mathbf{B}}^\infty=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\mathbf{B}^\infty(\omega t)\,\mathrm{d}t=0.$$