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On the existence of positive solution for second-order multi-points boundary value problems

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Abstract

Here we are concerned with the existence of positive solution for autonomous and nonautonomous second-order systems with multi-points boundary conditions. For nonautonomous systems we use the Schauder's fixed point theorem in a suitable Banach space, while for autonomous ones using fixed point theorems is usually useless because of the existence of trivial solution and for this we employed a method based on the implicit function theorem and topological degree. In order to verify the obtained results, we have considered some definite systems to verify the results numerically.

MSC: 34B10; 34B18; 34B15

Keywords: Ordinary differential equation; Boundary value problems; Topological degree

1. Introduction

The existence of solution for multi-points boundary value problems of Sturm–Liouvil kind of operators was initiated in [4,5]. As Erbe and Tang [1] showed, if the elliptic boundary value problem $-\Delta u = f(|x|, u)$ where $r < |x| < R$ is radially symmetric, then it can be converted into a Sturm–Liouvil problem. The above possibility motivated many authors to investigate in the existence of positive solutions for nonlinear second-order differential equations, see for example, Gupta [2,3], Palamides [9] and Ma [7]. A large part

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of the works made use the version of the Schauder fixed point theorem for a cone to prove the existence of positive solutions. In this paper, we consider autonomous as well as nonautonomous second-order systems. For nonautonomous systems, we have used the Schauder fixed point theorem to obtain the simple and easy verifiable conditions for the existence of a positive solution. For autonomous systems, as it is well known, using fixed point theorems may results in the trivial solution. For this, we developed a method based on the theory of topological degree to verify if the system has a positive solution. The method is easily applicable for a system of nonlinear second-order differential equations.

2. Nonautonomous system

Let us consider the following system:

$$x'' = f(t, x, x') \tag{2.1}$$

with the following boundary conditions:

$$\alpha x(0) - \beta x'(0) = 0, \quad x(1) = \sum_{i=1}^{m} \alpha_i x(t_i), \tag{2.2}$$

where α_i, α, $\beta > 0, f$ is a continuous function such that:

$$f(t, u, v) > 0, \quad u, v > 0, \quad t \in [0, 1] \tag{2.3}$$

$t_i \in (0, 1)$ and $m \geqslant 1$ is an integer. We also assume that

$$\sum_{i=1}^{m} \alpha_i t_i = 1 \tag{2.4}$$

which in accordance with the above assumptions

$$\sum_{i=1}^{m} \alpha_i t_i^2 < 1, \quad \sum_{i=1}^{m} \alpha_i > 1.$$

If we write the solution of Eq. (2.1) as

$$x(t) = B + At + \int_0^t (t - \tau) f(\tau, x(\tau), x'(\tau))\, \mathrm{d}\tau \tag{2.5}$$

then by the first boundary condition of the system we have $B = \frac{\beta}{\alpha} A$ and by the second one we get

$$A\left(\frac{\beta}{\alpha} + 1\right) + \int_0^1 (1 - \tau) f(\tau, x(\tau), x'(\tau))\, \mathrm{d}\tau = A \sum_{i=1}^{m} \alpha_i \left(\frac{\beta}{\alpha} + t_i\right)$$
$$+ \sum_{i=1}^{m} \alpha_i \int_0^{t_i} (t_i - \tau) f(\tau, x(\tau), x'(\tau))\, \mathrm{d}\tau.$$