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A fundamental solution method for three-dimensional viscous flow problems with obstacles in a periodic array☆

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Abstract

In this paper, we propose a fundamental solution method for three-dimensional viscous flow problems with obstacles in a periodic array. Our problem is mathematically a boundary value problem of the Stokes equation with periodic boundary conditions, to which it is difficult to give a good approximation by the ordinary fundamental solution method. Our method gives an approximate solution by a linear combination of the periodic fundamental solutions. In addition, we can compute the drag forces on the obstacles by using the data obtained in our method. Numerical examples for the problems of flows past spheres show the effectiveness of our method.

MSC: 65N35; 76D07

Keywords: Fundamental solution method; Viscous flow; Stokes equation; Periodic fundamental solution; Slice point method

1. Introduction

The fundamental solution method (or the charge simulation method) is a fast solver for problems of partial differential equations [12,11,17] and is widely used in science and engineering for the reasons that (i) it is easy to program, (ii) its computational cost is low and (iii) it achieves high accuracy under

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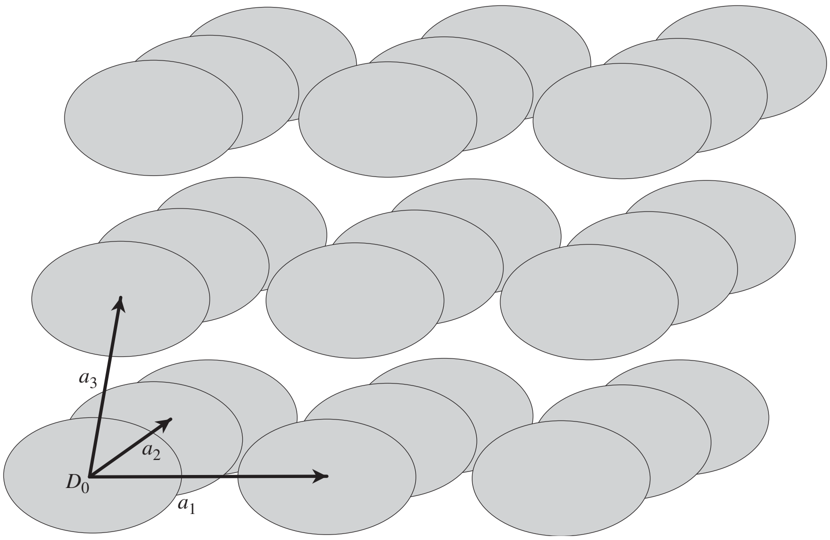

Fig. 1. Obstacles in a three-dimensional periodic array spanned by the vectors $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$.

some conditions. The method approximates the solution by a linear combination of the fundamental solutions.

In this paper, we are concerned with three-dimensional viscous flow problems with obstacles arranged infinitely in a three-dimensional periodic array (see Fig. 1) and propose a fundamental solution method for these problems. The problems of viscous flow past obstacles in a periodic array are important from theoretical and practical viewpoints, and have been studied by many authors. Hasimoto presented the periodic fundamental solution of the Stokes equation and applied it to the analysis of the viscous flow past an infinite array of spheres [4], whose results were improved by Sangani and Acrivos [16]. Ishii discussed three-dimensional Stokes flow problems with multiple planar arrays of small spheres in his paper [6], where he presented the periodic fundamental solution of the Stokes equation with a planar array of singularity points. Tamada and Fujikawa investigated the motion of steady two-dimensional viscous flow past an infinite row of circular cylinders based on the Oseen equation [18]. Liron investigated the Stokes flow due to an infinite array of Stokeslets parallel to flat planes or in a pipe with application to the analysis of fluid transport by ciliated organisms [7–9].

We assume that the flow is steady, incompressible and slow, that is, the Reynolds number is low and that a uniform force is acting on the whole fluid. Our problem is a boundary value problem of the Stokes equation, a linearization of the Navier–Stokes equation,

$$\mu \Delta \boldsymbol{v} - \nabla p + \boldsymbol{K} = \boldsymbol{0} \tag{1}$$

and the continuity equation

$$\nabla \cdot \boldsymbol{v} = 0, \tag{2}$$

where $\boldsymbol{v}(v_1, v_2, v_3)$ is the velocity of the flow, p is the pressure, $\boldsymbol{K}(K_1, K_2, K_3)$ is the uniform force per unit volume and μ is the viscosity. If we apply the fundamental solution method to this problem, it is