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Analysis of a predator–prey model with Holling II functional response concerning impulsive control strategy ☆

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Abstract

According to biological and chemical control strategy for pest control, we investigate the dynamic behavior of a Holling II functional response predator–prey system concerning impulsive control strategy-periodic releasing natural enemies and spraying pesticide at different fixed times. By using Floquet theorem and small amplitude perturbation method, we prove that there exists a stable pest-eradication periodic solution when the impulsive period is less than some critical value. Further, the condition for the permanence of the system is also given. Numerical results show that the system we consider can take on various kinds of periodic fluctuations and several types of attractor coexistence and is dominated by periodic, quasiperiodic and chaotic solutions, which implies that the presence of pulses makes the dynamic behavior more complex. Finally, we conclude that our impulsive control strategy is more effective than the classical one if we take chemical control efficiently.

Keywords: Holling II predator–prey model; Impulsive control strategy; Extinction; Permanence; Bifurcation

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1. Introduction

Pest outbreaks often cause serious ecological and economic problems. There are many ways to beat agricultural pests. Biological control is the reduction in pest populations from the actions of other living organisms, often called natural enemies or beneficial species (see [3,5,6]). Virtually all pests have some natural enemies, and the key to successful pest control is to identify the pest and its natural enemy, releasing the beneficial insect early when pest levels are low. One of the first successful cases of biological control in greenhouse was the use of parasitoid *Encarsia formosa* against the greenhouse whitefly *Trialeurodes vaporariorum* on tomatoes and cucumbers (see [25]). Another important method for pest control is chemical control. Pesticides are useful because they quickly kill a significant portion of a pest population and they sometimes provide the only feasible method for preventing economic loss. However, pesticide pollution is also recognized as a major health hazard to human beings and to natural enemies. Therefore, it is important to understand the life cycle of pest so that the pesticide can be applied when the pest is at its most vulnerable—the aim is to achieve maximum effect at minimum levels of pesticide.

Recently, in order to consider the consequences of spraying pesticide and introducing additional predators into a natural pest–predator system, many authors have suggested impulsive differential equations (see [2,11]) to investigate the dynamics of pest control model. Impulsive equations are found in almost every domain of applied science and have been studied in many investigations [1,7,9,10,12–24] in population dynamical systems. Ref. [15] developed Holling II functional response predator–prey system by periodic impulsive immigration of natural enemies. They gave the conditions for extinction of pest and permanence of the system and mainly studied the influence on the inherent oscillation caused by the impulsive perturbations. In Ref. [16], they presented and analyzed the pest–predator model under insecticides used impulsively and focused on the effects of the fraction of population which died due to the pesticide and the pulse period on the survive of the pest and predator. However, wherever possible, different pest control techniques should work together rather than against each other.

In Refs. [13,14], we constructed two kinds of predator–prey impulsive equations to model the process of periodic releasing natural enemies and spraying pesticide at fixed time, respectively. One is a classical Lotka–Volterra predator–prey impulsive system, which corresponding continuous system has a globally asymptotically stable positive equilibrium if it exists. The other is a predator–prey impulsive system with Holling I functional response, which corresponding continuous system may have a stable positive equilibrium and a stable limit cycle at the same time. In Ref. [14], from a biological point of view, we analyzed the dynamics of the system from two cases: general case (taking integrated pest management (IPM), that is, taking biological control and chemical control together) and special case (only choosing chemical control) and compared the validity of the IPM strategy with the classical methods (only biological control or chemical control). However, in Ref. [14], we ignored the side effects of pesticide on natural enemies and assumed the time of spraying pesticide and releasing natural enemies is the same. It is unreasonable. In Ref. [13], considering the effects of pesticide on natural enemies, we constructed a predator–prey impulsive system with Holling I functional response to model the process of periodic biological and chemical control at different fixed time. Since the unforced continuous predator–prey system with Holling I functional response has non-unique dynamics, that is, the solution of such system with different initial values either tends to a locally stable positive equilibrium or to a stable limit cycle, so in Ref. [13], we emphatically investigated the effects of the impulsive perturbations on the unforced continuous system and concluded that such impulsive system has different dynamic behaviors with