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# A $p$-version MITC finite element method for Reissner–Mindlin plates with curved boundaries

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**Abstract**

We consider the approximation of Reissner–Mindlin plates with curved boundaries, using a $p$-version MITC finite element method. We describe in detail the formulation and implementation of the method, and emphasize the need for a Piola-type map in order to handle the curved geometry of the elements. The results of our numerical computations demonstrate the robustness of the method and suggest that it gives near exponential convergence when the error is measured in the energy norm. For the robust computation of quantities of engineering interest, such as the shear force, the proposed method yields very satisfactory results without the need for any additional post-processing. Comparisons are made with the standard finite element formulation, with and without post-processing.


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## 1. Introduction

The Reissner–Mindlin (R–M) plate model is a widely used system of partial differential equations which describes the deformation of a thin plate subject to transverse loading. This two-dimensional

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model often replaces the full three-dimensional elasticity problem, when the thickness of the plate is small.

The numerical approximation of the solution to the R–M plate model has received much attention in recent years. Several techniques have been proposed to alleviate the two major computational difficulties associated with this problem, namely the presence of *locking* and *boundary layer* effects. The former occurs due to the inability of the approximating spaces to satisfy certain constraints imposed on the solution as the thickness $t$ of the plate tends to zero. The latter is due to the fact that the system of partial differential equations that describes the R–M plate model is singularly perturbed. The interplay of both phenomena is a rather complicated affair and the question of how to alleviate them *both* is still a mathematically open question (cf. [20]). Nevertheless, if one "separates" the two phenomena, then it is possible to design methods that yield very satisfactory results [25]. To deal with locking, there are two approaches one can take in the context of the finite element method (FEM): (i) enforce Kirchhoff's constraint exactly (by using, e.g., the high-order $p/hp$ versions of the FEM), or (ii) enforce Kirchhoff's constraint *weakly*, by using a modified variational formulation. To deal with boundary layers, the mesh has to be properly designed and in particular it should contain *thin* (anisotropic) elements along the boundary. If the proper mesh design is combined with the $p/hp$ version of the FEM, then exponential rates of convergence are possible.

Our goal in this article is to combine the above approaches, namely the $p/hp$ version of the FEM with a modified formulation, and extend their applicability to R–M plates with curved boundaries. In particular, we consider the so-called *Mixed Interpolated Tensorial Components* (MITC) elements, originally introduced in [8] in terms of the $h$ version of the FEM, and extended and analyzed in [22] in terms of the *hp* version. Even though in [22] the *hp* MITC method was defined for general curvilinear domains, the analysis was carried out only for straight-sided elements. Moreover, the only available numerical results showing the robustness of the *hp* MITC method are found in [2], and once more they are carried out only for straight-sided elements. (See also [3] for more on the approximation theory of *hp* MITC elements.) We wish to extend the results from [2] to the case of curved elements, and verify that the (original) definition of the *hp* MITC elements from [22] indeed works in practice when one deals with curvilinear domains. Building on the ideas used for nearly incompressible elasticity in [13], we are able to construct a $p$ version MITC method with the following properties:

- The method performs well, independently of the thickness of plate or the error measure used, provided one uses the proper mesh design for capturing the boundary layer that is (generally) present in the solution.
- No additional post-processing is required for the accurate calculation of quantities of engineering interest.
- Curved elements are handled with the use of a Piola-type mapping.

We hope that the present article will provide the groundwork for future research on these methods, especially in establishing the observed near exponential convergence rates.

In what follows, the usual $L^2$ inner product and norm are denoted by $(\cdot,\cdot)_\Omega$ and $\|\cdot\|_{0,\Omega}$, respectively, where $\Omega \subset \mathbb{R}^2$ with boundary $\partial\Omega$ smooth. The usual Sobolev spaces of functions on $\Omega$ with $r$ generalized derivatives in $L^2(\Omega)$ will be denoted by $H^r(\Omega)$, and their norms by $\|\cdot\|_{r,\Omega}$. Finally, the space $H_0^1(\Omega)$ will be used to denote functions in $H^1(\Omega)$ whose trace vanishes on $\partial\Omega$.