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Analysis of two frictional viscoplastic contact problems with damage☆

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Abstract

In this work we study two quasistatic frictional contact problems arising in viscoplasticity including the mechanical damage of the material, caused by excessive stress or strain and modelled by an inclusion of parabolic type. The variational formulation is provided for both problems and the existence of a unique solution is proved for each of them. Then a fully discrete scheme is introduced using the finite element method to approximate the spatial domain and the Euler scheme to discretize the time derivatives. Error estimates are derived and, under suitable regularity assumptions, the linear convergence of the algorithm is deduced. Finally, some numerical examples are presented to show the performance of the method.

Keywords: Damage; Friction; Viscoplasticity; Error estimates; Numerical simulations

1. Introduction

Situations of frictional contact abound in industry. Contacts of the braking pads with the wheel or the tire with the road are just a few simple examples. Because of its importance, the engineering literature concerning this topic is rather extensive (see, e.g., [17,24,31,36] and references therein).

In many industrial problems involving frictional contact there is a need to take into account the damage of the material, due to mechanical stress or strain. Early models, derived from the thermodynamical principles, were introduced in [18,19], where some numerical simulations were performed. Recently, damage models have been improved and studied in many engineering papers (see, e.g., [2,3,14,26–29,35] and references therein).

Viscoelastic and viscoplastic contact problems have been studied in recent papers. In monograph [22] some quasistatic contact problems arising in viscoplasticity and viscoelasticity were described, but the damage of the material was not taken into account. Quasistatic contact problems, involving viscoelastic and viscoplastic materials and including the effect due to the damage, were studied in [7,9,10,16,21] and a dynamic viscoelastic contact problem with damage

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was considered in [8]. This paper extends the results provided in [11] to the case with damage, including a numerical algorithm to solve the fully discrete problem and performing some numerical simulations, and it is parallel to [7].

In this work, two frictional contact problems involving viscoplastic materials and damage are considered. In the first one, the contact is bilateral and it is modelled by Tresca's law while, in the second one, the contact is modelled with a simplified version of Coulomb's law of dry friction. We perform the variational analysis of these problems, including the existence of a unique solution to the models by using fixed point arguments. Our interest is also the numerical analysis of the models including numerical simulations. An algorithm is developed, by regularizing the frictional term, for the numerical solution of the fully discrete problem.

The paper is structured as follows. In Section 2 the mechanical problems are stated together with the assumptions on the data and their variational formulations. The existence and uniqueness of the solution of the models are provided in Section 3. Then, in Section 4 a numerical scheme is introduced, based on the finite element method to approximate the spatial domain and the forward Euler scheme to discretize the time derivatives. Error estimates are deduced for the approximative solutions and, under suitable regularity assumptions, the linear convergence of the algorithm is obtained. Finally, some numerical simulations are presented in Section 5.

2. Mechanical and variational problems

We denote by S_2 the space of second-order symmetric tensors on $\mathbb{R}^2$. Let "$\cdot$" be the inner products on $\mathbb{R}^2$ or S_2, and $|\cdot|$ the Euclidean norms on these spaces.

Let us consider a viscoplastic body that occupies the domain $\Omega \subset \mathbb{R}^2$, and let the time interval of interest be $[0, T]$, $T > 0$. The outer surface $\Gamma = \partial\Omega$ is assumed to be Lipschitz continuous, and is divided into three disjoint measurable parts Γ_D, Γ_F and Γ_C. For a.e. $\boldsymbol{x} \in \Gamma$, we denote by $\boldsymbol{\nu}(\boldsymbol{x})$ and $\boldsymbol{\tau}(\boldsymbol{x})$ the unit normal and tangential vectors outward to Γ, respectively. A density of volume forces $\boldsymbol{f}_B$ acts in Ω and surface tractions of density $\boldsymbol{f}_F$ are given on Γ_F. The body is assumed to be clamped on Γ_D, and so the displacement field vanishes there. Finally, the body is assumed to be in contact with a foundation on the contact surface Γ_C (see Fig. 1).

We denote by $\boldsymbol{u}$ the displacement field, $\boldsymbol{\sigma}$ the stress tensor and $\boldsymbol{\varepsilon}(\boldsymbol{u})$ the linearized strain tensor. Moreover, let ζ be the damage field, which is defined in Ω and measures the density of the microcracks in the material. The material is assumed viscoplastic with the following constitutive law (see, e.g., [13,15,22,23] and references therein):

$$\dot{\boldsymbol{\sigma}} = \mathscr{E}\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}) + \mathscr{G}(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}(\boldsymbol{u}), \zeta), \tag{1}$$

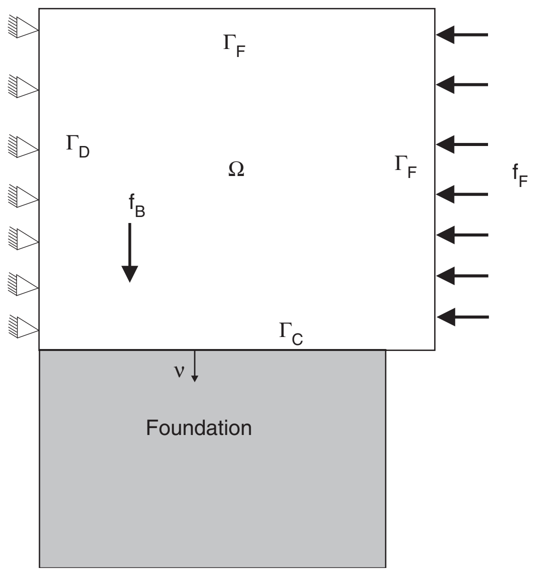

Fig. 1. A viscoplastic body in frictional contact.