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Image denoising using principal component analysis in the wavelet domain

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Abstract

In this work we describe a method for removing Gaussian noise from digital images, based on the combination of the wavelet packet transform and the principal component analysis. In particular, since the aim of denoising is to retain the energy of the signal while discarding the energy of the noise, our basic idea is to construct powerful tailored filters by applying the Karhunen–Loéve transform in the wavelet packet domain, thus obtaining a compaction of the signal energy into a few principal components, while the noise is spread over all the transformed coefficients. This allows us to act with a suitable shrinkage function on these new coefficients, removing the noise without blurring the edges and the important characteristics of the images. The results of a large numerical experimentation encourage us to keep going in this direction with our studies.

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Keywords: Wavelet packets; KL transform; Filter banks; Recursive matrices; Image denoising

1. Introduction

Signals and images are often corrupted by noise in their acquisition or transmission. Let the noise model be

$$\bar{f} = f + \xi,$$

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where $\bar{f}$ is the noisy image, f is the original image and ξ is i.i.d. Gaussian noise with mean zero and standard deviation σ. The goal of a denoising method is to remove the noise, while retaining the important signal features as much as possible. Traditionally, in the existing literature there are two kinds of denoising methods; namely, linear and nonlinear techniques. Linear denoising methods, such as Wiener filtering [14], are simple and cheap to implement. However, they sometimes tend to blur the proper edge structure of an image, which determines a good visual quality. In order to preserve the real characteristics of a signal, a vast literature has recently emerged on signal and image denoising using nonlinear denoising techniques, such as, e.g., the well-known *Wavelet Shrinkage* introduced in [11]. This method is based on filtering the wavelet coefficients in the frequency detail subbands by means of the *Soft Thresholding* operator, which takes the coefficients of absolute value larger than the threshold and shrinks them by the threshold value towards zero.

More precisely, the wavelet expansion of $\bar{f} \in L_2(\mathbb{R}^2)$ is considered, namely

$$\bar{f} = \sum_{j\in\mathbb{Z}^2,\ k\in\mathbb{Z},\ \psi\in\Psi} \bar{c}_{j,k,\psi}\psi_{j,k} \quad \text{with } \bar{c}_{j,k,\psi} = \int_{\mathbb{R}^2} \bar{f}(x)\psi_{j,k}(x)\,\mathrm{d}x,$$

where the functions $\psi \in \Psi$ are the two-dimensional wavelets, constructed via the tensor product of one-dimensional wavelet and scaling functions [8,10]. Hence, the Soft Thresholding filter acts on the noisy wavelet coefficients $\bar{c}_{j,k,\psi}$ in the following way:

$$S_\lambda(\bar{c}_{j,k,\psi}) = \begin{cases} \mathrm{sign}(\bar{c}_{j,k,\psi})(|\bar{c}_{j,k,\psi}| - \lambda), & |\bar{c}_{j,k,\psi}| > \lambda, \\ 0, & |\bar{c}_{j,k,\psi}| \leqslant \lambda. \end{cases}$$

The wavelet expansion of the denoised image is therefore given by

$$f^\star = \sum_{j\in\mathbb{Z}^2,\ k\in\mathbb{Z},\ \psi\in\Psi} S_\lambda(\bar{c}_{j,k,\psi})\psi_{j,k}.$$

In recent years, many papers have been devoted to the wavelet denoising problem, by proposing many alternative choices both for the thresholding rule and the shrinkage parameter λ (see, for example, [2,15,13,16]). In particular, in [16], a first attempt has been proposed to combine linear Wiener filtering and wavelet thresholding in an unique nonlinear denoising method. The idea discussed in [16] is to estimate the covariance matrix of each block of wavelet coefficients and to use its eigenvalues to obtain a more efficient thresholding rule.

On the other hand, an interesting aspect of wavelet thresholding algorithms is that they can lead to lossy compression of the starting data. Actually, in the existing literature, many works addressed a strong connection between lossy compression and denoising, especially with nonlinear algorithms (see [6,7]). Therefore, a good compression method can provide a suitable model for distinguishing between signal and noise.

In this paper, we exploit the analogy between denoising and lossy compression problems in order to introduce a new denoising method, which combines the good properties of the wavelet packet analysis with those of principal component analysis, realized using the Karhunen–Loéve (KL) transform in the wavelet domain. In fact, the KL transform is a well-established data-dependent tool for image and signal compression, but its great computational complexity highly reduces its field of application. Our idea is to evaluate the covariance matrix of the wavelet packet coefficients and to exploit the magnitude of its eigenvalues to make a decision about the amount of information contained in each coefficient. In fact, the