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# Joint probability generating function for a vector of arbitrary indicator variables

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## Abstract

We obtain formulas for the probability generating function of general multivariate Bernoulli distributions, and for the moment generating function of the aggregate claim amount for individual risk models with dependencies. Several examples are given.


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## 1. Introduction

Let us consider an individual risk model in a given time interval, consisting of $n$ risks or portfolios with corresponding total claim amounts $X_i$, $i = 1, \ldots, n$. We assume that

$$X_i = \begin{cases} B_i & \text{if } I_i = 1, \\ 0 & \text{if } I_i = 0, \end{cases} \tag{1}$$

where $I_i$ is a Bernoulli random variable with mean $p_i$, and $B_i$ is a positive random variable. In actuarial applications $p_i$ is small and is interpreted as the probability that the $i$th policy produces a positive claim

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$B_i$. The aggregate claim amount

$$S = X_1 + \cdots + X_n$$

represents the total loss, and is a measure of riskiness of the portfolio. Much work has been devoted to the computation of the distribution function of $S$ for the case of independent $\{B_i, X_i,\ i = 1, \ldots, n\}$ (see e.g. [16,18] and the references therein).

However, recent research has shown that, in many cases, dependencies between risks lead to underestimation of the stop-loss premium for $S$ (see e.g. [13,6,7,2,22,1,3–5,9], and the references therein).

In [3] expressions for the moment generating function of $S$ are given for some special models of portfolios with risks of form (1). Cossette et al. [3] propose a dependence structure for individual risk models where occurrence of risks is based on a common mixing variable $\Theta$, and evaluate numerically the impact of such risks dependencies. They consider the following dependence:

$$P(I_i = 1|\Theta = \theta) = 1 - r_i^{\theta}, \quad P(I_i = 0|\Theta = \theta) = r_i^{\theta}, \tag{2}$$

where $r_1, \ldots, r_n \in [0, 1]$ are fixed parameters of the model. Hence, each random variable $I_i$ is influenced by the values of $\Theta$, and given $\Theta = \theta$, the conditional probability of no occurrence of claim is a decreasing function of $\theta$. Under the assumption that $(I_1 = 1|\Theta = \theta), \ldots, (I_n = 1|\Theta = \theta)$ are independent random variables, Cossette et al. [3] obtained the moment generating function of $(X_1, \ldots, X_n)$, namely

$$M_{(X_1,\ldots,X_n)}(t_1, \ldots, t_n) = \int_0^{\infty} \prod_{i=1}^{n} [r_i^{\theta} + (1 - r_i^{\theta})t_i]\, \mathrm{d}F_{\Theta}(\theta), \tag{3}$$

where $F_\Theta$ is the distribution function of $\Theta$.

In [4] the model described by (1) is considered. The insurance portfolio is divided into $m$ different classes, where each class $j$ contains $n_j$ risks, $j = 1, \ldots, m$, and the total number of risks is $n = n_1 + \cdots + n_m$. The dependencies between risks are given in terms of the “occurrence random variables” $I_{jk},\ j = 1, \ldots, m,\ k = 1, \ldots, n_j$, in the following way. Assume that

$$I_{jk} = \min(J_{jk} + J_j + J_0, 1), \tag{4}$$

where $J_{jk}, J_j$, and $J_0$ are independent Bernoulli random variables with corresponding expectations $1 - \overline{p}_{jk}, 1 - \overline{p}_j$ and $1 - \overline{p}_0$. These random variables correspond, respectively, to individual, class, and global risk factors.

Cossette et al. [4] obtain the following expression for the moment generating function of the aggregate claim amount $S = \sum_{j=1}^{m}\sum_{k=1}^{n_j} X_{jk}$:

$$\begin{aligned} M_S(t) = \overline{p}_0 &\left[ \prod_{j=1}^{m} ((1 - \overline{p}_j) \prod_{k=1}^{n_j} M_{B_{jk}}(t) + \overline{p}_j \prod_{k=1}^{n_j} (p_{jk} + (1 - p_{jk}) M_{B_{jk}}(t)) \right] \\ &+ (1 - \overline{p}_0) \prod_{j=1}^{m} \prod_{k=1}^{n_j} M_{B_{jk}}(t), \end{aligned} \tag{5}$$

where $p_{jk} = \overline{p}_0\, \overline{p}_j\, \overline{p}_{jk}$.